THE OPPORTUNITY COST HYPOTHESIS AND THE CYCLICAL BEHAVIOUR OF RESEARCH SPENDING

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ABSTRACT. Schumpeter's memorable aphorism of "creative destruction" has spawned numerous important contributions in the study of economic growth but there has been substantially less attention paid to his hypothesis that R&D expenditure should display counter-cyclical features. I develop a theoretical model which addresses that claim through processes of both innovation and imitation which drive growth in the model and endogenously determine market structure. Here, a measure of aggregate R&D expenditure, comprised of both innovation and imitation, displays a positive contemporaneous correlation with aggregate output and a more disaggregated variable representing individual firms' research intensities displays a pro-cyclical behaviour following shocks to productivity and aggregate demand, which is in agreement with the empirical evidence. However, a re-interpretation of the original meaning of the opportunity cost hypothesis is possible by looking at the ratio of research spending to output, which displays clear counter-cyclical features.

Keywords: research & development, endogenous growth, cycles.

JEL Classification: E32, E44, O31, O32, O40
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1. Introduction

A famous hypothesis originally formulated by Schumpeter, [33] makes the argument that recessions are ideal opportunities to invest in research that would yield new growth, and profit, generating technologies. The logic is simple: idle resources in a downturn can be reallocated to inventive activity at a lower cost than at any other point in the cycle. Thus, downturns provide excellent opportunities to invest resources in productivity enhancing technologies.

Despite this, both empirical and theoretical contributions have since supplied ample evidence to suggest R&D expenditure or indeed effort very clearly displays pro-cyclical behaviour. This is entirely at odds with the assumption that a lower opportunity cost during recessions should incentivise efforts to discover new technologies which could then be profitably exploited in periods of expansion.

This paper first briefly summarises a few key empirical findings that show pro-cyclical behaviour of research spending in both aggregate and disaggregated data, as well as those theoretical contributions that attempt to explain the reasons why it fails to corroborate the Schumpeterian hypothesis. It then develops a Schumpeterian model of endogenous growth in which entrants rely on venture capitalists to finance innovative research expenditure, while established incumbents attempt to adopt new inventions so as to remain in the market. The underlying structure of the model is that of a standard real business cycle model, which allows a more detailed analysis of the cyclical profile of research expenditures in the presence of different shocks to aggregate output. These shocks are based on the three main explanations for pro-cyclical R&D in the literature, namely aggregate demand, total productivity and credit availability.

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1 Measured as the fraction of labour resources devoted to research activity.
2 The literature refers to these models as 'quality ladders'. In a previous paper [34], I discuss three competing approaches that attempt to integrate these models in a real business cycle framework.
2. Literature

2.1. Empirical Evidence. Figures 1 and 2 establish a *prima facie* argument for what appears to be a positive correlation between real GDP and aggregate research expenditure. Indeed, while figure 2 looks at the growth rates for both of these variables in the sampled period, figure 1 shows their deviations from the trend, calculated using a Hodrick-Prescott filter. The same conclusion is immediately apparent in both graphs, and indeed correlation between deviations in aggregate R&D and in real GDP rounds up to 52.8%, while the equivalent for the growth rates in both variables is 43.9%. Countercyclical behaviour in aggregate research spending is simply not a feature of the data.

More importantly, though, recent research (Barlevy, [3], Comin and Gertler[4], [7], Fatás, [11], Ouyang, [27], [28], Walde and Woitek, [37]) has collected significant evidence contra the Schumpeterian hypothesis, instead postulating that R&D is strongly pro-cyclical at the aggregate and industry level. One of the explanations put forward in order to explain the cyclical profile of research spending that it is technology shocks, by raising the value of innovations through increased productivity of ideas, driving increases in R&D. This explanation is based on the cyclical behaviour of profits: should these display a strongly pro-cyclical bias, it will likely force research spending to follow the same pattern despite the counter-cyclical bias implied by the opportunity cost hypothesis.

Similarly, changes to aggregate of industry-wide demand can have similar effects: by affecting the cyclical pattern of profits, it similarly offsets the counter-cyclical opportunity cost effect by changing the value of firms and, therefore, the optimal amounts of research. This type of explanation is not based on a departure from the opportunity cost hypothesis, but of ”dynamic externalities” (Barlevy, [3]) that cause entrepreneur to behave in a shortsighted fashion or demand-side variability as is argued in Geroski and Walters, [16]. The argument for a demand-pull explanation of research investment relies on upward shifts in demand increasing the expected profitability of innovations and,

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3The latter argue that this is evident not only in higher frequencies but also medium-term frequencies, as low medium-term waves of economic growth tend to reflect on relatively low R&D effort.
A competing theory to both hypotheses based on aggregate shocks having implications for the volatility of profits, and, hence, of firms’ valuations, is the notion that despite there being a strong incentive to engage in research spending during downturns, there may be a scarcity of available funds during these periods. That is: if access to credit or financial markets is restricted during periods of economic contraction, it follows that even though firms and entrepreneurs would very much like to expand the rubric of research expenditure at the expense of resources devoted to production, the reality is such that they cannot obtain the necessary funding in order to do so. Hence, some authors maintain that the opportunity cost hypothesis does not hold because of the restricted availability of credit. Aghion, et al., [2], argue that absent credit constraints, empirical evidence from France suggests private R&D is actually counter-cyclical and that, even allowing for credit constraints, there is an asymmetric response of R&D to economic conditions, in that firms decrease expenditure sharply during recessions as credit tightens but do not increase it accordingly during periods of recovery. This insight is corroborated in Aghion, et al., [11] using a simple model that implies an increase in the pro-cyclical behaviour of R&D when there are constraints on the availability of credit. Additional research by Mancusi and Vezzulli, [24], present evidence that credit constraints have a ”a significantly negative effect on the probability to set up R&D activities”.

2.2. Theoretical Contributions. There have been several recent attempts to integrate Schumpeterian growth dynamics with the standard RBC model, but as early as Pelloni, [30], there had been attempts to reconcile business cycle dynamics with endogenous acquisition of knowledge, chiefly through learning by doing or other human capital investments. Contributions in this vein have been revised to accommodate Schumpeterian features, as in Maliar & Maliar, [23], but these authors find that while their model predicts highly pro-cyclical innovations, their cycle-generating mechanism (the shifting of

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4Arguments along those lines can be extended to any type of shock that increases the equilibrium or steady-state value of the firm: even if the substitution effect decreases R&D expenditure, the increase in firm value more than compensates for it and it may well be expected to increase, thus seemingly invalidating the opportunity cost hypothesis.
resources between R&D and production) implies that R&D is counter-cyclical, much in
the vein of the opportunity cost hypothesis. This is at variance with what the empirical
evidence mentioned previously suggests. A variety of theoretical models, such as Bean, [4],
Canton, [6], and Ozlu [29], corroborate the counter-cyclical behaviour of human capital
accumulation, which in turn agrees with empirical evidence mentioned by these author.
The usefulness of models that use human capital as a proxy for innovative activity and
as a guide to how R&D responds to output variability is, however, compromised by the
fact that it displays a counter-cyclical behaviour, a variable that has been shown to have,
at the very least, moderately pro-cyclical behaviour.

To make this point clearer, in Maliar and Maliar, [23], the authors outline what is
essentially a model of human capital accumulation in which investments in human capital
are thought of as a form of R&D spending. That being the case, it is hard to reconcile
the evidence of pro-cyclical R&D with counter-cyclical human capital investments. Thus,
the clarity of models that conflate the two is hampered significantly while models that rely
on some form of human capital investment or learning-by-doing do not yield meaningful
predictions about the behaviour of R&D over the cycle.

Hence, a theoretical model attempting to provide a rationale for the behaviour of re-
search expenditure time series must explicitly describe decisions to invest in innovative
activity by firms, consumers or a social planner. There is significant variety in these,
meaning that their predictions might not be comparable across models, but all provide
reasonable and workable approximations to the task of exploring the role of economic
cycles in a growth model with creative destruction.

Walde, [36], suggests a reasonably straightforward approach in the form of a two sector
canonical endogenous growth model. Rather than rely on exogenous shocks, he isolates
the long term trend in the relevant variables from potential sources of short run distur-
bances. This enables the author to show that R&D would display a pro-cyclical behaviour.

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5 The authors explicitly describe it as an R&D variable.
6 Sakellaris and Spilimbergo, [32], present evidence suggesting that human capital is in fact counter-
cyclical for OECD countries while credit constraints seem to account for pro-cyclical behaviour for non-
OECD countries.
However, under relaxed assumptions, the same author in Wälde, [35], finds that R&D may indeed display counter-cyclical behaviour, dependent on certain assumptions on the functional forms implied. That suggests that developing a theoretical model in which research spending can display either pro-cyclical or counter-cyclical is feasible, and that the question then becomes one of which better mimics the behaviour of real world data.

One particularly interesting contribution is that of Phillips and Wrase, [31], who, despite not significantly improving on the performance of the standard RBC model in terms if matching relevant second moments, come close to mimicking important features of the data. They do so despite no labour-leisure trade-off and with a very simplistic description of all non-Schumpeterian aspects of the economy. This suggests that, with appropriate extensions and a richer model namely one in which there is a more complex mechanism of technology diffusion this approach has significant potential. This relates to a prominent feature of the aforementioned Maliar and Maliar, [23], namely, the presence of persistence in output despite no assumptions of rigidity in prices or consumption paths, which again outlines the potential of integrating Schumpeterian growth models and RBC models. While, Phillips and Wrase, [31], fail to establish that the process of 'creative destruction' is a reasonable source of technology fluctuations, they nevertheless succeed in developing an approach that allows for an analysis of how exogenous technology shocks affect the cyclical behaviour of expenditure on innovation; an approach which is partially replicated in these pages. Other authors, such as Fatás, [11], developed models in which a variable aggregate demand, via shocks to employment, is the main source of fluctuations. This particular model highlights the effect of fluctuations in demand in the profitability of innovations, which in turn affect firms optimal choices of research effort and are the reason why R&D exhibits pro-cyclical behaviour.

Comin and Gertler, [7], argue that are medium-term cycles in a number of relevant macro variables and, significantly for our discussion, that R&D exhibits pro-cyclical behaviour not only at higher frequencies but also in the medium term. That is, apart from a positive correlation with fluctuations in output in the short-run, R&D seems to follow what the authors claim to be medium-term (32 to 200 quarters) cycles. They rely, however, in an exogenous shock in households labour-leisure choice following evidence
suggesting that this is the most important source of cyclical variation. Even though their model yields a wide variety of predictions consistent with the data, the authors argue that inclusion of a process of diffusion of technology has scope to greatly improve the performance of the model provided technology adoption is pro-cyclical. This general framework, expanded in Comin, Gertler and Santacreu, [8], it is a significant improvement on preceding work, and is very similar in structure to the approach pursued herein. This approach is again closely linked to work developed by Galo, [13] and [14], the latter of which includes one of the first attempts in the literature to incorporate credit restrictions in a model with both endogenous growth and cycles.

2.3. Summary. In order to capture the cyclical behaviour of aggregate research expenditure in a model that is calibrated to real world data and that yields predictions that are validated by preceding research, the present construct introduces a variety of changes to state-of-the-art models in endogenous growth with exogenous fluctuations. The Schumpeterian dynamics are refined through a process of innovation and imitation discussed at length in Sérôdio, [34], which allows for a significant refinement of the entry/exit dynamics absent in most models in this category. Similarly, it enables the replication of the correct dynamics in price mark-ups, profits and degree of competitiveness in the market.

In turn, these improve the ability of the model to match key features of the data, meaning any conclusions regarding the response of R&D to various shocks is substantially more reliable. Further, it allows the inclusion of the three different sources of fluctuations identified in the literature as relevant for the ‘unexpected’ pro-cyclical behaviour of expenditure in innovative activity and technology dissemination.

3. Model

The basic outline of the model is a discrete time version of a standard Schumpeterian growth model, in which growth occurs through stochastic improvements in the quality of the intermediate good. There are, however, a few important departures from the usual framework. Instead of the benchmark Dixit-Stiglitz model of imperfect competition, all

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7 See Hall, [18], for more detail.
8 Bilbiie, Ghironi, and Melitz, [5] provide further arguments for the inclusion of these departures from the standard model.
firms in each sector are assumed to produce and sell an identical good and to compete à la Cournot. The immediate result is that market power comes not from selling a different variety of the good over which the firm holds a patent but from the fact that there is a finite number of firms in the market for that specific sectoral good at each point in time. One implication is that, in the limit, each good’s price could, conceivably, be equal to the price that would prevail in a competitive setting should the number of firms approach infinity. Thus, the model allows for a full range of potential market structures, from monopoly to perfect competition.

Tied to this initial departure is the process through which innovation and technology acquisition occurs. The Arrow effect ensures that, absent any specific advantages that incumbents may possess in the innovative process, challengers will always outbid the former when competing for resources to devote to innovation; which implies that all innovation happens through entrepreneurial effort. Technological progress occurs when an entrant successfully improves on the current quality level of the good being produced.

Incumbents are not, however, helpless in the face of creative destruction: in the absence of well defined intellectual property rights, as soon as a new technology is developed, they can attempt to reverse engineer it and reenter the market in order to compete on an equal footing with the successful entrepreneur, should they be successful. This process is assumed to be instantaneous and occurs right before production takes place, meaning that any and all incumbents that are able to replicate the new level of quality will remain in the market, while those who fail to do so lose their market position and see their position and firm value liquidated.

Entrants are then assumed not to be able to imitate a new technology developed\footnote{Indeed, while there is a continuum of these, by virtue of there being a continuum of entrepreneurial households, I assume in the equilibrium section that they are all identical and, therefore, behave exactly like a representative entrepreneur would, rendering the issue of copying one’s own invention irrelevant. Nevertheless, the assumption that copying is impossible unless in possession of the technology that has just been displaced avoids that potentially complicated result.} by one of their own. In short, only incumbents, who possess the technology that has been just displaced, are able to reproduce the technology just discovered by an entrepreneur.
This assumption resonates with the widely held perception that the majority of innovations are come by through the efforts of innovative outsiders, rather than well established firms attempting to cement their market position. It also validates the perception that despite not being as innovative, firms can remain at the technological frontier if they are sufficiently adept at adopting new innovations and bringing to market equivalent products, thereby competing with innovators on an equal footing.

3.1. Households. The representative working household faces a standard problem of maximising a time separable utility function over consumption of a composite final good and leisure. Supplied labour is hired out by final goods firms at the wage rate \( w_t \). In addition to this, the representative consumer must also choose her preferred amount of holdings of loans to \( (A^h_t) \) the entrepreneurial households, as well as physical capital (which depreciates at rate \( \delta \) ) that is rented out to intermediate goods firms. Being partly owned by these households, a fraction of firm profits are paid directly to these agents. The optimisation problem for the \( h^{th} \) working households can be written as follows:

\[
\max_{(c^t, l^t)} U = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \exp(v_t) \log c^h_t - \psi \log(1-l^h_t) \right\}
\]

\[
\text{s.t.} \quad w_t l^h_t + \pi^h_t + (1 + d_t - \delta)k^h_t + R^A_t A^h_t = c^h_t + k^h_{t+1} + A^{h+1}_{t+1}
\]

Entrepreneurial households face a very different problem. They do not earn any labour income as their only activity concerns the production of new ideas that can be transformed into successful innovations and, consequently, firms. Consumption smoothing is achieved by issuing ‘persona’ bonds purchased by the working households.\(^{10}\) Entrepreneurs earn a stochastic income, \( y^*_t \), which will be define further on and choose an optimal consumption path. Thus, the optimisation problem for the representative household in this group is:

\(^{10}\)While this may sound odd, it is no different from assuming perfect credit markets with respect to consumption financing. The model could easily incorporate government bonds and/or a credit market with built-in frictions, but these would be distractions from the aim of analysing how credit restrictions to innovative activity, specifically, affect optimal research expenditure. Focusing on how entrepreneurs succeed in smoothing consumption in the absence of a steady income stream such as wages would distract from that primary objective.
 Aggregate demand shocks are modelled in the spirit of Nakajima, [26]:

\[
\max_{\{c_t\}_{t=0}^{\infty}} U = E_t \sum_{t=0}^{\infty} \beta^t \{\exp(v_t) \log c_t^e\}
\]

s.t. \( y_t^e - A_{t+1}^e + R_t^A A_t^e = c_t^e \)

(3) \[ v_t = \rho v_{t-1} + \varepsilon_{v,t} \]

Given perfect capital markets and the assumption of homogeneous agents in the working household, the aggregate stock of capital can be modelled evolving according to the standard equation\(^{11}\):

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]

3.2. Production.

3.2.1. Final good. Final output is a composite of a unit mass of sectoral goods, each with a different degree of technological sophistication. Each sector produces a single, undifferentiated good and there is a finite number of producers, \( n_t(m) \) in each of them:

\[
Y_t = \exp(u_t) \left[ \int_0^1 \left( \omega_t(m) q_t(m) \int_0^{n_t(m)} y_t^e(m) \frac{\mu - 1}{\mu - 1} dm \right) \frac{\mu}{\mu - 1} dm \right]
\]

Where \( \omega_t(m) \) is a weight assigned to each sectoral quality level that takes the form\(^{12}\):

\[
\omega_t(m) = \left( \int_0^1 q_t(m) dm \right)^{-\frac{1}{\mu}}
\]

The choice of shocks to neutral aggregate productivity rather than labour enhancing productivity is related to how it may generate a substitution effect that would potentially generate a counter-cyclical behaviour in research expenditure: higher labour productivity

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\(^{11}\)For simplicity, the model abstracts from investment costs and other frictions related to the cost of capital or the cost of investment goods.

\(^{12}\)This weight ensures aggregate output is linear in the average quality of the sectoral goods.
would lead to both higher output and a relative decrease in firm’s willingness to spend on research. Neutral productivity shocks ensures that the opportunity cost hypothesis isn’t validated by assumption on the nature of economy-wide technology shocks. These shocks obey the following law of motion:

\[ u_t = \rho u_{t-1} + \varepsilon_t^u \]

Each sectoral good has a different degree of technological sophistication and, therefore, \( q_t(m) \) varies across all sectors. Aggregating quality across all sectors yields:

\[ Q_t = \int_0^1 q_t(m) dm \quad (6) \]

Similarly, each sectoral output is produced by \( n_t(m) \) identical firms who compete à la Cournot. This means that instead of there being a single monopolist in each sector, there is now a fixed number of firms that compete in selling good \( m \). In turn, output for firm \( j \) is given by the following production function:

\[ y_j^t(m) = [k_j^t(m)]^\alpha [l_j^t(m)]^{1-\alpha} \quad (7) \]

The assumption of competition in quantity, à la Cournot, yields a price mark-up that is a function of the price elasticity of the sectoral output as well as the number of firms in each market. This closely follows the approaches developed in Galí & Zilibotti, [12].

\[ p_t(m) = \frac{\mu n_t(m)}{\mu n_t(m) - 1} \quad (8) \]

The implication is clear: as the number of firms in sector \( m \) increases, it tends to the competitive outcome of a zero mark-up over marginal cost. Under the assumption that sectoral output is evenly divided between all market participants, we can write each individual firm’s profit function as:

\[ \text{A more detailed exposition of how these price mark-ups can be generated, including Bertrand and Stackelberg versions can be found in Etro, [10].} \]
Each firm must solve a two-step optimisation problem. First, it must choose the level of inputs that minimises costs relative to a fixed revenue. Additionally, it must also choose an optimal choice of output given the industry-wide price:

\[
\min_{k_t(m), l_t(m)} \bar{p}_t(m)y_t^2(m) - k_t^2(m)d_t - l_t^2(m)w_t \\
\text{s.t. } \bar{p}_t(m)y_t^2(m) \leq m
\]

3.3. **Research.** The economy-wide average quality level is given by \(Q_t\), which was derived in equation (6). Innovation in sector \(s\) occurs when an entrant is successful in developing a better quality version of the good produced in that sector. The likelihood that any given challenger is successful is given by:

\[
\text{Prob}(\text{challenger } e \text{ is successful}) = \eta(z_e^r(m), a_t(m))
\]

This probability depends on the quality-adjusted level of expenditure by innovators, as well as the efforts developed by venture capitalists in order to transform an idea with potential (created by the entrepreneur) into a worthwhile concern. The expected value for the average aggregate quality level, then, is given by:

\[
\mathbb{E}_t[Q_{t+1}] = \lambda \left[ \int_0^1 (\eta(z_e^r(m), a_t(m))) \, dm \right] Q_t + \left( 1 - \int_0^1 [\eta(z_e^r(m), a_t(m))] \, dm \right) Q_t
\]

The logic here is straightforward: the expected level of quality in period \(t + 1\) is given by the sum of the probability that any of the entrepreneurs is successful in developing a new innovation, which increases the quality level by \(\lambda\); and the probability that none of them achieves that aim. In turn, and as mentioned previously, the cost of research for the entrepreneur per unit of quality is \(z_e^r(m)\), which means that total research expenditure for each agent of this type is:

\[^{14}\text{The Arrow replacement effect implies that incumbents will never choose to innovate themselves.}\]
Whenever an entrepreneur is successful, the incumbent can remain in the market if he successfully copies the innovator’s new technology. The probability that he is accomplishes this is given by:

\[
\text{Prob(incumbent } j \text{ is successful)} = \phi(z_j^t(m))
\]

Where \( z_j^t(m) \) is total research expenditure on imitation by the \( j^{th} \) firm in the \( m^{th} \) sector. All of this implies that the value of an existing firm must obey the following dynamic optimisation problem for the incumbent:

\[
r_t V_j^t(q_t(m), n_t(m)) = \max_{z_j^t(m)} \left\{ \pi_j^t(q_t(m), n_t(m)) - z_j^t(m)q_t(m) + \phi(z_j^t(m)) \left[ \int_0^{n_t(m)} \phi(z_j^t(m)) \left( V_{t+1}^j(\cdot) - V_j^t(\cdot) \right) - (1 - \phi(z_j^t(m))) V_j^t(\cdot) \right] \} \]

The number of firms in any given sector changes whenever there is a successful discovery: all incumbents lose initially lose their market share and are replaced by a challenger, which is subsequently joined in production by whichever number of firms that succeed in adopting the technology. Hence, the law of motion for the total number of firms in each sector is given by the following expression:

\[
E_t(n_{t+1}(m)) = \eta(z_t(m), a_t(m)) \left( 1 + \int_0^{n_t(m)} \phi(z_j^t(m)) \right) + (1 - \eta(z_t(m), a_t(m))) n_t(m)
\]

### 3.4. Venture capital financing.

Entrepreneurial households lack the resources to engage in innovative activity, so they must turn to external financing to acquire the necessary financial means. There is a single venture capitalist, owned by the blue collar households, which finances all research activity by purchasing a stake \( s \) in any future new firms created by the entrepreneurs. This venture capitalist is assumed to be risk neutral on the basis that, by financing research in all sectors, she is insulated from idiosyncratic risk in each
specific research venture.

The price of each stake in a future business, $s$, is given by the initial amount made available by the VC to each potential entrepreneur:

$$b^*_t(m)q_t(m)$$  \hspace{1cm} (17)

This initial financing is proportional to the quality level in that sector. It stands to reason that a prospective entrepreneur would, all things equal, demand a higher price for a fraction of any future firm. The entrepreneur then uses these funds to finance his research activity, which have a cost of $z^*_t(m)$ per unit of quality, meaning that total investment in R&D by a single entrepreneur is given by:

$$z^*_t(m)q_t(m)$$  \hspace{1cm} (18)

That yields, as previously discussed, a probability of innovation that is a function of both the research expenditure of the entrepreneurs as well as the support of the venture capitalists. It is assumed to take the following functional form:

$$\eta(z^*_t(m), a_t(m)) = \eta(z^*_t(m))^{1-\gamma} (a_t(m))^\gamma$$  \hspace{1cm} (19)

This probability of innovation, however, depends also on the input from the venture capitalist, who apart from providing the funds must also engage in expenditure of its own to provide adequate input into the project in order for it to succeed. Without it, potential entrepreneurs lack the expertise required to transform an important idea into a feasible operation. The investment decision and optimal choice of effort expended by the venture capitalist are decided separately in a two-stage optimisation problem that can be described as follows.

The entrepreneur’s optimisation problem is characterised, at the first stage, by a choice of $s$, the share of the firm’s value that accrues to the venture capitalist, and $b^*_t(m)$, the amount which the venture capitalist is prepared to pay for said share. Once these have been successfully negotiated, both agents choose optimal amounts of research effort $a^*_t(m)$
and \( z_t^e(m) \).

Thus, the entrepreneur’s optimisation problem can be characterised as follows\(^{15}\):

\[
\Omega_t = \max_{s,b_t^e(m)} \{ y_t^e(m) - z_t^e(m)q_t(m) \}
\]

with

\[
y_t^e(m) = \eta(\cdot)(1 - s)E_t [V_{t+1}(q_{t+1}(m), n_{t+1}(m))] + b_t^e(m)q_t(m)
\]

s.t.: 

1. \( b_t^e(m)q_t(m) \leq \exp(\chi_t) \{ \eta(\cdot)sE_t [V_{t+1}(q_{t+1}(m), n_{t+1}(m))] - \theta a_t(m)q_t(m) \} \)
2. \( z_t^e(m)q_t(m) \leq b_t^e(m)q_t(m) \)
3. \( \eta_a(z_t^e(m), a_t(m))sE_t [V_{t+1}(q_{t+1}(m), n_{t+1}(m))] = \theta_t q_t(m) \)
4. \( \eta_z(z_t^e(m), a_t(m))(1 - s)E_t [V_{t+1}(q_{t+1}(m), n_{t+1}(m))] = q_t(m) \)

Exogenous shocks, \( \exp(\chi_t) \), to the availability of credit, i.e., the amount each venture capitalist makes available to a potential entrepreneur, obey the following law of motion:

\[
\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + \varepsilon_t^\chi
\]

The interpretation of this parameter is important. A likely parallel can be immediately thought of in the guise of the stochastic loan-to-value (henceforth LTV) ratio in the financing structure of mortgages or corporate loans. In these environments, \( 1 - \text{LTV} \) can be immediately thought of as the “proportional cost of collateral repossession for banks given default”\(^{15}\). Essentially, it represents a wedge between the expected future valuation of the asset and the principal financial outlay plus interest, and will typically be a value either moderately or substantially lower than one, depending on the degree of leverage, with parity the case of maximum leverage.

---

\(^{15}\)This formalisation of the relationship between entrepreneur and venture capitalist closely follows that described in Keuschnigg (2004). The present structure removes the constraint that the expected gains attributable to the entrepreneur must exceed wage income because it is assumed that there is no mobility between the entrepreneurial class and blue collar workers.
In this context, however, there are a few significant departures from that standard scenario. Where those environments ordinarily attempt to model the behaviour of commercial banks, this paper focuses on the role of venture capitalists. These, instead of lending a certain amount and earning interest on the loan, choose to purchase a participation of size \( s \) in a future enterprise. Thus, there is no interest accrued. Additionally, instead of a long-run or steady-state LTV of less than one, I choose a value of one so as to ensure zero profits in equilibrium for the venture capitalist. A positive shock to this variable would then imply that the equivalent to the LTV ratio in the model would go above one, which can be interpreted as an overtly optimistic valuation of the enterprise on the side of the venture capitalist and, therefore, represents a loosening on the financial constraints that entrepreneurs must respect. Evidently, a positive shock means the venture capitalist loses money on the enterprise.

This is clear by restricting attention to the venture capitalist’s one period income at time \( t \):

\[
y_t^f = \eta(z_t^e(m))s \mathbb{E}_t [V_{t+1}(q_{i+1}(m), n_{i+1}(m))] - \theta a_t(m)q_t(m) - b_t^e(m)q_t(m)
\]

Evidently, if \( b_t^e(m)q_t(m) \) exceeds the expected value of the venture capitalist’s share in the firm net of his own investment in the research stage of the project, then she incurs in a loss.

Regardless of success in innovating, an entrepreneur will always receive the amount \( b_t^e(m) \), which is paid to him in advance by the venture capitalist. Should he be successful in creating a new company, he will retain a fraction \( 1 - s \) of the total capitalisation of the company. In order to innovate, however, he must expend a total amount of resources \( z_t^e(m)q_t(m) \).

The entrepreneur’s optimal decision must also respect constraints 1 to 4. The first constraint is the participation constraint by the venture capitalist: unless his returns exceed the initial financial layout plus his own research effort, he will not be willing to finance the project and it therefore breaks down. The second constraint imposes the restriction that the entrepreneur cannot borrow directly from white collar households to finance research effort: at most, he can use the total amount made available by the venture capitalist. The
final two constraints can be interpreted as incentive constraints: once the initial contract has been drawn up, they must be respected to ensure that neither participant slacks off - this is particularly relevant for the entrepreneur, who might choose instead to keep the amount paid by the venture capitalist and not engage in research effort.

Following [22], the solution can be found by backward induction. Optimal choices of effort are reached when the contract between the two parties has already been agreed upon, which means that both $s$ and $b_t^f(m)$ will have been chosen. Therefore, the venture capitalist will attempt to maximise his own surplus by deciding upon the optimal amount of his own research effort, $a_t(m)$:

$$
\max_{a_t(m)} \{ \eta (z_t^e(m), a_t(m)) s \mathbb{E}_t [ V_{t+1}(q_{t+1}(m), n_{t+1}(m))] - a_t(m) \gamma_t q_t(m) \}
$$

(23)

Which results in the incentive constraint (3), which must hold with equality. The first constraint, the participation constraint for the venture capitalist, might still hold with a strict inequality, however, which would mean that the financier would reap a positive profit from the project. That is only possible if he attempts to negotiate, on the initial stage, a less beneficial deal for the entrepreneur - which would induce the latter to exert less effort and the final constraint would not be obeyed. Therefore, the deal negotiated between entrepreneur and financier will be such that the venture capitalist does not reap any benefit from the enterprise, i.e., the first constraint will hold with equality.

Solving for the optimal amount of effort from the venture capitalist, we get the following relationship between VC effort and research effort by the entrepreneur:

$$
a_t(m) = \frac{\epsilon_{\eta,a} z_t^e(m)}{1 - \epsilon_{\eta,a} \theta_t}
$$

Here it is important to note that the venture capitalist’s optimal choice of research intensity crucially depends on the optimal amount of research effort by the entrepreneur. Having defined these, it is now useful to turn to the initial stage of the project to determine the share each agent ends up with as well as the initial financial outlay by the financier.
Constraint (2) holds with equality, which means that the entrepreneur exhausts the entirety of the resources made available to him by the venture capitalist, who in turn negotiates that price so as to induce optimal behaviour by the former. Equally, the shares negotiated should, in principle, reflect the contribution each agent’s input to the probability of a successful outcome.

In order to determine the equilibrium share, however, we must make assumptions about what each agent knows. If the probability of an innovation and each agent’s contribution are known to both, then we can use constraints (3) and (4) to determine the optimal shares, which are then given by:

\[ s^* = \frac{\epsilon_{n,z}}{1 - \epsilon_{n,a} + \epsilon_{n,z}}; \quad 1 - s^* = \frac{1 - \epsilon_{n,a}}{1 - \epsilon_{n,a} + \epsilon_{n,z}} \]

3.5. Equilibrium. In addition to assuming a market structure for each sector in which all firms are identical\(^{16}\) I further restrict the analysis to a fully symmetrical equilibrium in which all sectors are assumed to be identical and at the same level in terms of quality. Additionally, given that most variables grow along the equilibrium path, these are transformed according to \( \tilde{x}_t = \frac{x_t}{q_t} \), so that the model and equilibrium conditions only have stationary variables.

The standard Euler and capital accumulation equations are modified to include potential growth rate for the level of quality\(^{17}\) and although there are two different types of household, consumption for both behaves identically and we can therefore collapse the two separate Euler equations into a single one:

\[
\beta (1 + r_{t+1}) c_t \exp(v_{t+1}) = (1 + g_{t+1,t}) c_{t+1} \exp(v_t) \tag{24}
\]

\(^{16}\)Which is implicit in the assumption of Cournot competition.

\(^{17}\)This growth rate is also the balanced growth path growth rate for output, consumption, capital and investment. It is important to note that deviations in this potential growth rate do not necessarily translate into higher output growth; e.g., a temporary shock might increase per capita output in the short run and simultaneously reduce investment in research, thereby reducing the potential growth rate. Once both return to the steady-state level, only permanent changes to R&D expenditure have an effect on the growth rate of per capita output.
While the aggregate capital accumulation is, once having transformed the variables into stationary ones:

\[ k_{t+1}(1 + g_{t+1,t}) = (1 - \delta)k_t + i_t \]  

Labour supply can also be derived from the first order conditions of the blue collar household, yielding a familiar expression:

\[ \psi \frac{1}{1 - l_t} c_t = w_t \exp(\nu_t) \]  

From the intermediate firm’s optimal choice of inputs, we can easily derive demand for capital and hours worked, which yields the following equations:

\[ \alpha y_t = \frac{\mu n_t}{\mu n_t - 1} (r_t + \delta)k_t \]  
\[ (1 - \alpha) y_t = \frac{\mu n_t}{\mu n_t - 1} w_t l_t \]

Given the assumptions required to determine the equilibrium outcomes, the production function is reduced to:

\[ y_t = \exp(\nu_t)k_t^{\alpha} l_t^{1-\alpha} \]

From the research sector, we can derive the free entry condition that determines the optimal amount of imitation effort, which is conditional on the amount of research expenditure by potential entrants.\[ \]  

\[ \phi'(z_t^i) \eta(z_t^e, a_t) \frac{\mu n_t}{n_t} y_t = r_{t+1} + \eta(z_t^e, a_t) \]

As shown above, the optimal amount of imitation effort by each incumbent is, in equilibrium, related to the research efforts of innovators: a higher fraction of innovative R&D expenditure by potential entrants leads to higher imitation efforts: a higher probability of displacement by a challenger leads to more investment by incumbents attempting to retain their position in the market.\[ \]
expenditure will mean a higher probability of any given incumbent is replaced, which in motivates higher imitation effort from the incumbents. Bargaining between entrepreneurs and venture capitalists determines the optimal ownership structure of the firm as well as the equilibrium amount of financial resources made available to the innovators:

\[
\phi'(z_t^e)(b_t + \theta a_t) = s
\]  
(31)

\[
z_t^e \leq b_t \exp(\chi_t)
\]  
(32)

The financier’s optimisation program then determines the amount of input she will put into each research project, which in turn will be a function of the entrepreneur’s optimal research efforts. This is summarised as follows:

\[
a_t \theta_t = z_t^e
\]  
(33)

The equilibrium mechanics are straightforward: an exogenous shock that increases the value of a firm will lead to an increase in the amount entrepreneurs are willing to spend in research. Because half of any new firm will then be owned by the venture capitalist, this also increases that she will make available to potential entrants. In addition to increasing the amounts of available funds, the venture capitalist will also increase her own expenditure amounts in response to the increase in the representative entrepreneur’s expenditure. Finally, a higher probability of innovation means the likelihood the current crop of incumbents will be displaced is higher, which will induce higher expenditure in imitation from firms trying to stay in the market.

From the law of motion for the quality level, we can derive the equilibrium growth rate of real output along the balanced growth path for this economy:

\[
g_{t+1,t} = (\lambda - 1)\eta(z_t^e, a_t)
\]  
(34)

19As highlighted in the previous section, the assumption of constant returns to scale in the probability of innovation means that each party gets half of the firm, regardless of how productive each contribution is to the final outcome.
Knowing optimal innovation and imitation expenditure, we can write the equilibrium expression, or law of motion, for the expected number of firms one period ahead:

$$E_t n_{t+1} = \eta(z^e_t, a_t) \left(1 + n_t \phi(z^j_t)\right) + (1 - \eta(z^e_t, a_t)) n_t$$

(35)

Aggregate research expenditure is the sum of both innovation and imitation expenditure, where total imitation effort if the sum of the individual efforts of each incumbent:

$$z_t = z^e_t + n_t z^j_t$$

(36)

The economy’s resource constraint is then:

$$y_t = c_t + i_t + z_t + \theta_t a_t$$

(37)

Finally, the three laws of motion for the three kinds of shocks are as follows:

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \quad \text{Productivity shock}$$

$$\nu_t = \rho_v \nu_{t-1} + \varepsilon_{v,t} \quad \text{Demand shock}$$

$$\chi_t = \rho_\chi \chi_{t-1} + \varepsilon_{\chi,t} \quad \text{Liquidity shock}$$

(38) (39) (40)

4. Model simulations

In this section I will focus on detailing the various sources for the data used to calibrate the model, as well as outline the basic mechanisms and sources of propagation in the model. One important caveat applies in all discussions of the results: all key variables in the model are transformed to ensure stationarity. Therefore, when discussing the co-movements between research & development and aggregate output, that is equivalent to discussing the relationship displayed by their non-stationary equivalents. On the other hand, when interpreting changes to a single transformed variable, the same cannot be said of the non-stationary equivalent: i.e., a decrease in the transformed real output in the model may not translate into a decrease in actual real output, because it may be associated to an increase in the level of technological advancement: which correlates to an increase in output.
Nevertheless, the usual interpretations are entirely valid for transformed variables and valid inferences may be drawn from the relationships uncovered by the model.

4.1. **Calibration.** The parameters in the model can broadly be divided into standard ones that are frequently used in the literature and parameters that are specific to the innovation and imitation probabilities. The former include the capital share $\alpha$, which is set to 0.36; the wage markup $\psi$, set at 1.5; the discount factor $\beta$, set at 0.96; the depreciation rate $\delta$, chosen to be 0.06, the persistence parameter $\rho$ for the various shocks, set to 0.95 for all; and the standard deviation of all the shocks, again all of the same size, $\sigma$, set to 0.016. All additional parameters are chosen so that the growth rate of output along the balanced growth path is 2.2%, which is the rate of growth of per capita GDP in the US in the period between 1960-2010; and so that the ratio of total research & expenditure to per capita GDP matches the average value for this ratio during that period.

These nonstandard parameters include the elasticity of the probability of innovation and elasticity of the probability of imitation, $\gamma$, which are assumed to be identical and set at 0.5; the jump in quality $\lambda$, set to 1.1 (implying a jump in quality of 10% whenever an innovation arrives); the elasticity of demand for sectoral goods $\mu$, set at 6; and, finally, the parameters $\eta$ and $\phi$, set to 34 and 7.62 respectively. Interpreting these last two parameters is not straightforward, but either can be thought of as affecting the marginal productivity of an additional unit of spending, $\eta$ for the research efforts of entrants and $\phi$ for the outlay of the incumbents. Therefore, these numbers aren’t very significant in themselves, but when combined with the steady-state values for expenditure by either type of agent, they determine the probabilities of successful innovation and imitation along the balanced growth path. The simulations described here imply a balanced growth path probability of innovation of 22.2% while the probability of imitation is much higher at 53.2%, implying that there is a 1 in 2 chance for each individual incumbent that she might succeed at copying a new invention along the balanced growth path.

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20 The choice of a unit elasticity of labour supply restricts the degrees of freedom in this regard.
21 Details of the values these take can be found in the annex.
22 Evidence presented in [7] and [17] indicates that an elasticity of new patents with respect to expenditure in research and development of 0.5 is a reasonable estimate.
23 Meaning that every period there is roughly a 1 in 5 chance that a successful innovation will occur.
There is a dearth of empirical evidence for the probability of successfully developing a new innovation, and what little there is focuses on specific industries, which means these estimates may lack a degree of generality. \[21\] report a mean frequency of product innovation of 26% in their sample, while \[19\] report a success rate of 13% for products that are new to the market in the financial sector in the UK. Finally, data from the pharmaceutical industry presented in \[20\] suggests the historical cumulative rate of success for a new drug to go through all the stages of the trial process is 14%.\[24\] The value of 22% for the probability of success of an innovation along the balanced growth path in the simulated fits well within this range, albeit edging towards the upper bound.

Estimates for imitation are even harder to come by, but again in \[19\], Heffernan, Fu and Fu report a measure of successful innovations that are new to the firm but not new to the market, i.e., innovations that are being newly adopted by firms despite already being available to competitors that has a mean of 28.4% in their sample. The estimates for innovation shown previously indicate that there may be some downward bias in the success rate for each new product, but evidence on the likelihood of imitation or the fraction of new products that are imitated is fairly limited. Cooley & Yorukoglu, \[9\], argue, with evidence lifted from \[25\], that 71% of innovations are eventually imitated. This would place the probability in our simulations in between these estimates and therefore, we can admit it as a fairly reasonable number.\[25\]

4.2. Data. The model is calibrated using yearly per capita real output data from 1960 to 2009 taken from the data archive of the Federal Reserve Bank of St. Louis, meaning that every period in the model is equivalent to a year in the data. Furthermore, quarterly data on the percentage shares of gross domestic product for the same period was taken from the Business for Economic Analysis, which allows us to create individual series for the various rubrics, such as consumption, investment and government expenditure. Finally, data on research & development expenditure was collected from the National Science Foundation statistical output on total, private and public, R&D expenditure in the US for the same period.

\[24\]The authors note that this rate seems to have fallen dramatically to 8% in recent years.

\[25\]Such a wide range of estimates in the literature does not allow us a greater degree of accuracy in determining what actual rates of imitation or likelihood of imitation may be.
Given the absence of government expenditure in the model and the inclusion of R&D under private investment and government outlays rubrics respectively for private and public research expenditure, further changes to the data are required. Instead of subtracting government from total output, public expenditures were divided, using data made available by the BEA, into capital and consumption expenditures, with the former being added to the private investment series and the latter to household consumption. This leads to discrepancies between the steady-state ratios of consumption to output and private investment to output, mostly because these were required in order to match the rate of output growth along the balanced growth path and long term ratio of R&D expenditure to output. Finally, NSF data allows us to differentiate between private and public expenditure in research, which means both are subtracted from private and public expenditure series respectively, with a new variable being created that displays the behaviour of aggregate R&D outlays.

5. Discussion

The results in terms of the de-trended, stationary model, are unambiguous: aggregate R&D expenditure is pro-cyclical in response to both productivity or demand shocks and counter-cyclical when the source of disturbance is an increase in the availability of credit. These results require a substantial degree of additional scrutiny, however, particularly because they apply only to the de-trended model.

5.1. Productivity shock. A productivity increase (figures 3 and 4) makes the process of production of the final good temporarily more effective, thereby raising the profits and value of a firm above their trend value as output increases relative to its stationary level too. Firms respond by hiring more labour and driving wages up, while higher demand for capital drives up both investment and the real interest rate. Consumption and income both rise, and this increase in affluence makes venture capitalists more likely to increase the availability of resources to potential entrepreneurs: this, combined with the higher value of firms makes entrepreneurs’ research efforts increase as well. Finally, with both
higher profits and a higher likelihood of being displaced by a challenger, incumbents increase the amount of resources devoted to research into successfully imitating the new technologies. This also implies that, with more innovation and more effort by incumbents to remain in the market, there is an increase in the overall degree of competitiveness - leading to a fall in the price mark-up.

A different picture emerges, however, once we take into consideration the path of the actual time-series of the variables. Given that the model is transformed in order to allow for a stationary solution, it is hard to evaluate claims of pro-cyclicality fully, given that most expenditure variables are likely to evolve in the same direction, particularly in the case of shocks whose primary impact is through firm profits and value. Reconstructing the time path of output per capita and aggregate R&D allows us to calculate the share of the latter in the former for the three different shocks: figure 9 shows this ratio falling after a productivity shock, which can be interpreted as aggregate research expenditure displaying counter-cyclical features. Figure 10 provides additional evidence in favour of this interpretation, with the ratio of R&D expenditure to total investment (which is calculated as the sum of research expenditure and physical capital investment) falling after a positive productivity shock. The picture is even clearer when we disaggregate this category expenditure even further into three different sources: innovative expenditure, imitation (incumbent) expenditure and market size. The latter, as mentioned earlier, is strongly pro-cyclical (figure 15), and is therefore an important source of potential bias in aggregate spending. As for the two individual measures figures 13 and 14, both very clearly display counter-cyclical behaviour.

5.2. **Aggregate demand shock.** The shock to aggregate demand displays very similar behaviour in terms of the stationary variables, with all behaving identically to the case when the source of the shock is productivity. The only major difference lies with the real wage rate, which is entirely expected, given the shift in preferences towards consumption means workers are willing to accept lower rates with which to finance the increase in expenditure. Again, the shares of aggregate, imitation and innovation expenditure follow the same pattern as in the previous case, with all of them displaying counter-cyclical

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26 The ratio of each of these over aggregate output.
behaviour. The only exception in this instance being the share of aggregate research expenditure over total investment expenditure\(^{27}\), which increases in response to the demand shock. The reason why it should is clear: while a productivity shock increases the marginal product of capital, a demand shock only does so indirectly, and so the increase in investment is obviously lower.

Taking stock, the behaviour of the variables of interest in the model allows for a tentative reconciliation of the Schumpeterian opportunity cost hypothesis with the sweeping empirical evidence pointing of pro-cyclical R&D expenditure. Namely, shocks to productivity and demand affect firm value positively, and that leads to increases in aggregate expenditure across the board, to which research expenditure is not immune. It does, however, proportionally increase less than total output (and investment in physical capital, in the case of productivity shocks), which can be interpreted as validating, at least partially, the opportunity cost hypothesis - the nature of these shocks means that investments yielding returns only in the future are less appealing when output is higher. The case of demand shocks warrants a caveat: because there is higher preference for consumption, this item of expenditure is preferred on aggregate to either investment or R&D, and so the moderate increase in the ratio of research expenditure to investment has a less clear interpretation than in the case of the productivity shock.

5.3. **Liquidity shock.** The final source of fluctuations analysed takes the form of an exogenous variation in the *pseudo*-loan-to-value ratio of the net worth of an unrealised innovation. An increase in this value means venture capitalists relax the binding constraint on the amount they are willing to part with up front for a share in potential new concern, and can thus be interpreted as an increase in liquidity that allows entrepreneurs to spend a larger fraction of real resources on innovative activity.

The results are puzzling at first glance. Relaxing the constraint on the availability of resources decreases total production in the economy, in the process reducing consumption as well as investment in physical capital. Lower production means less labour demand, which drives down wages enough that, over the cycle, there is an increase in the amount

\(^{27}\)Again, this is defined as the ratio of \(z_t\) over the sum of \(i_t\) and \(z_t\).
of hours worked relative to the long run trend. Finally, innovation and imitation effort display a strongly counter-cyclical behaviour, with sharp increases that are entirely to be expected, given the relative easing of financial constraints. Incumbent expenditure (imitation effort) is curtailed by below trend firm values throughout the cycle that represent a much lower incentive for established firms to attempt to retain their position in the market in the face of creative destruction. The overall effect remains positive, however, by virtue of the still overriding threat of losing their place in the market. Looking at the shares of the different research expenditure items over aggregate output largely corroborates the counter-cyclical nature of this spending, with figures 9 and 12 showing increases in each of the relevant aggregate measures over the duration of the effect of the shock, with figures 13 and 14 showing those results mimicked for imitation and innovation expenditure.

A final word on the role of credit in the financing of research investment. While it has been suggested that restrictions on the availability of resources may be an important reason why firms don’t invest more in research during recessions, this is very different from those disturbances originating in the availability of credit in this market. Indeed, financial resources made available by venture capitalists and their optimal amount of effort are strongly correlated, and the response of this variable tells an important story of what happens to credit depending on the source of the shock. Indeed, figures 4 and 6 show that we should expect it to behave in a very pro-cyclical manner, while an increase in the availability of resources is of the opposite sign of the variation in output relative to its trend. Figure 18 replicates the counter-cyclical behaviour of both kinds of research expenditure when we consider the ratio between the time path of this variable and the time path of output.

It remains a somewhat baffling conclusion that relaxing constraints to the availability of credit should reduce output in the short run, but it is important to consider here that

\[28\] The argument could be made that this effect is asymmetrical, with credit being a constraint mostly during recessions. If credit constraints are relaxed during output expansions, this should not affect the decision of firms attempting to cut down on research spending. That said, the constraint may still be binding even during periods of strong growth if the optimal level of expenditure from the point of view of the innovators is still far from what the financial sector is willing to supply. I largely abstract from these considerations, but will therefore explain the processes at work mostly with the example of a recession.

29
this describes the behaviour of the stationary output variable, which has been de-trended for purposes of tractability. Indeed, by looking at the behaviour of the ratio of output to its trend (in which no shocks occur), a different picture emerges. There is a very small reduction in real output at time 0, by virtue of research expenditure reducing the levels of both private consumption and investment, but this is quickly reversed because the 'long-run' growth rate of technology is also temporarily higher, given the increase in innovative research and venture capitalist efforts. Indeed, while all shocks lead to a permanently higher level of output\(^29\), real output will, in the long-run, be significantly higher in the case of a liquidity shock than in the case of a demand shock, and will be only slightly below the level generated through an exogenous productivity shock. This is because shifting resources to demand will reduce the amounts invested in innovation without a significant gain in terms of output, as is the case when a technological shock is considered.

6. Conclusion

Since Schumpeter first argued for the 'cleansing' effect of recessions, the idea that investment in research and development responded primarily to its counter-cyclical opportunity cost has been thoroughly rejected from an empirical standpoint and instead researches have focused on attempting to explain exactly why we should expect firms to behave in a way that contradicts that elementary logic. Thus, the mechanism and its underlying reasoning have been mostly rejected as accurate representations of how firm level and aggregate R&D behave along the economic cycle, by virtue of its lack of empirical validity.

Despite that, in providing a theoretical framework that highlights the role of shocks in driving firm profits and value as important reasons why research expenditure fails to behave as predicted by the opportunity cost hypothesis, subsequent authors have ignored important aspects of how both firm level and aggregate level representations of such spending behave. Namely, their share in aggregate output and with respect to total investment often respond to positive shocks in a way that is entirely compatible with a variation of the hypothesis. Indeed, taking these effects into consideration, I present a

\(^{29}\)Recall that, in an absolute sense, research efforts increase in response to any positive shock. This leads to temporarily higher rates of 'long-run' growth that are made permanent through the faster development and incorporation of new technologies.
version of this argument implying that although this item of expenditure clearly displays pro-cyclical behaviour by virtue of how shocks affect the profitability and value of firms, its importance in aggregate income is commensurate with the interpretation that there is indeed an opportunity cost motive which can be said to affect optimal R&D spending.

The role of financial constraints, and specifically those pertaining to the availability of resources to finance innovative activity warrant more careful discussion: when shocks originate in technological developments or fluctuations in aggregate spending, they behave according to what past research would predict, varying pro-cyclically and further relaying that behaviour to spending on research. On the other hand, when they originate in the availability of resources to entrepreneurs, the general equilibrium outcome implies a short-lived reduction in output by virtue of resources being syphoned off from consumption and investment into R&D, thus driving down income. Empirically, then, this an important question for future research: fluctuations in the availability of funds due to conditions in the overall macroeconomy are expected to be pro-cyclical and to generate pro-cyclical research spending; fluctuations originating in the financing of research itself are expected to lead to counter-cyclical research, thus reinforcing the opportunity cost hypothesis in a setting in which the profits and firm value channel is absent.
References


# 7. Appendix

## Table 1. Deep parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>( \alpha )</td>
<td>0.36</td>
<td>Capital share</td>
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</tr>
<tr>
<td>( \rho )</td>
<td>0.95</td>
<td>Shock persistence</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.016</td>
<td>Size of shock</td>
</tr>
<tr>
<td>( \phi )</td>
<td>7.62</td>
<td>-</td>
</tr>
<tr>
<td>( \psi )</td>
<td>1.5</td>
<td>-</td>
</tr>
</tbody>
</table>

## Table 2. Steady state

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>0.81†</td>
<td>0.7488</td>
</tr>
<tr>
<td>( i )</td>
<td>0.16†</td>
<td>0.2185</td>
</tr>
<tr>
<td>( z )</td>
<td>0.025††</td>
<td>0.0251</td>
</tr>
<tr>
<td>( z^j )</td>
<td>-</td>
<td>0.0082</td>
</tr>
<tr>
<td>( z^n )</td>
<td>-</td>
<td>0.0077</td>
</tr>
<tr>
<td>( g )</td>
<td>0.022†</td>
<td>0.0220</td>
</tr>
<tr>
<td>( n )</td>
<td>-</td>
<td>2.1355</td>
</tr>
</tbody>
</table>
Table 3. Calibration: standard deviations

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(dy_t)</td>
<td>1.5448(^{†})</td>
<td>1.3089</td>
</tr>
<tr>
<td>(dc_t)</td>
<td>1.4146(^{†})</td>
<td>0.7550</td>
</tr>
<tr>
<td>(di_t)</td>
<td>4.9849(^{†})</td>
<td>3.4598</td>
</tr>
<tr>
<td>(dz_t)</td>
<td>1.6770(^{††})</td>
<td>1.4693</td>
</tr>
<tr>
<td>(dz_j)</td>
<td>–</td>
<td>1.7361</td>
</tr>
<tr>
<td>(dz^n_t)</td>
<td>–</td>
<td>1.0590</td>
</tr>
<tr>
<td>(dn_t)</td>
<td>0.4684(^{†})</td>
<td>0.1058</td>
</tr>
<tr>
<td>(dg_t)</td>
<td>1.6960(^{†})</td>
<td>1.0590</td>
</tr>
</tbody>
</table>

Table 4. Calibration: correlations with output

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>((dy_t, dc_t))</td>
<td>0.8456(^{†})</td>
<td>0.9443</td>
</tr>
<tr>
<td>((dy_t, di_t))</td>
<td>0.6722(^{†})</td>
<td>0.9694</td>
</tr>
<tr>
<td>((dy_t, dz_t))</td>
<td>0.5281(^{††})</td>
<td>0.9694</td>
</tr>
<tr>
<td>((dy_t, dz_j))</td>
<td>–</td>
<td>0.9845</td>
</tr>
<tr>
<td>((dy_t, dz^n_t))</td>
<td>–</td>
<td>0.8060</td>
</tr>
<tr>
<td>((dy_t, dn_t))</td>
<td>0.0815(^{†})</td>
<td>0.6612</td>
</tr>
<tr>
<td>((dy_t, dg_t))</td>
<td>0.5626(^{†})</td>
<td>0.8060</td>
</tr>
<tr>
<td>((dy_t, dp_t))</td>
<td>–</td>
<td>–0.6612</td>
</tr>
<tr>
<td>((dy_t, dz_z_{i+1}t))</td>
<td>–</td>
<td>–0.9043</td>
</tr>
</tbody>
</table>

Table 5. Variance decomposition

<table>
<thead>
<tr>
<th>Variables</th>
<th>Productivity shock</th>
<th>Demand shock</th>
<th>Financial shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_t)</td>
<td>90.23</td>
<td>9.76</td>
<td>0.00</td>
</tr>
<tr>
<td>(c_t)</td>
<td>66.40</td>
<td>33.56</td>
<td>0.04</td>
</tr>
<tr>
<td>(z_t)</td>
<td>86.19</td>
<td>8.34</td>
<td>5.48</td>
</tr>
<tr>
<td>(z_j)</td>
<td>89.55</td>
<td>8.67</td>
<td>1.78</td>
</tr>
<tr>
<td>(z^n_t)</td>
<td>60.18</td>
<td>5.83</td>
<td>34.00</td>
</tr>
<tr>
<td>(n_t)</td>
<td>90.23</td>
<td>8.16</td>
<td>1.61</td>
</tr>
</tbody>
</table>
Figure 1. Deviations in real GDP and aggregate R&D

Figure 2. Growth in real GDP and aggregate R&D
Figure 3. Impulse response functions for a shock to $u_t$. 
Figure 4. Impulse response functions for a shock to $u_t$
Figure 5. Impulse response functions for a shock to $\nu_t$
Figure 6. Impulse response functions for a shock to $v_t$. 
Figure 7. Impulse response functions for a shock to $\chi_t$
Figure 8. Impulse response functions for a shock to $\chi_t$
Figure 9. Aggregate R&D share of GDP per capita

\[ \frac{z_t}{z_t + x_t} \] following a shock to productivity
Figure 11. $\frac{z_t}{i_t + z_t}$ following a shock to demand

Figure 12. $\frac{z_t}{i_t + z_t}$ following a shock to liquidity
Figure 13. Individual firm R&D share of GDP per capita

Figure 14. Innovative R&D share of GDP per capita
Figure 15. Number of firms

Figure 16. Price mark-up
**Figure 17.** Ratio between GDPpc after each shock and trend GDPpc

![Graph showing the ratio between GDPpc after each shock and trend GDPpc.](image1)

**Figure 18.** VC expenditure share of GDP per capita

![Graph showing the VC expenditure share of GDP per capita.](image2)