A MODEL OF GENERAL EQUILIBRIUM PRICING OF INNOVATION WITH EFFORT EXERTING CONSUMERS

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Abstract

This paper reviews the theoretical literature on the subject of innovation and intellectual property rights, highlighting specific concerns that have been raised by authors at least since Schumpeter [29]. We also propose an extension of the formulation envisioned by Boldrin-Levine [6] and Quah [20] where consumers are allowed some discretion in the effort levels they exert while accessing the innovative good. The main results imply that when consumers are not allowed to reap any benefits from the costly effort, i.e., there is excessive appropriability by the firm, the decentralised market outcome is not equivalent to the social optimum. We also discuss possible further extensions to this formulation and argue that even though the market outcome is not socially efficient, lack of appropriability by innovators need not be a concern in a competitive economy.

1. Introduction

Traditional economic analysis on the subject on innovation or inventive activity is deeply rooted in the seminal works of Arrow [1], Nelson [16] and Schumpeter [29]. While through different approaches, each of them helped define contemporary economic discourse around the issue of innovation and the production of ideas or "'intellectual goods"'. Subsequent research throughout the late 70's and 80's cemented the view that patents were essential instruments in achieving social efficiency, with major contributions such as [12], [14], [15], [8] and [22], among others.

While both of these lines of enquiry followed starkly different methods[1], they seem to unambiguously point out that an unfettered market is too prone to failure for innovation to be carried out in a socially efficient fashion. Furthermore, some of the arguments have seeped into the so-called "new growth theory" (as documented by Romer in [24], [25] and [26]), in which concepts such as nonrivalry and excludability take central stage, thereby lending even more weight to the widespread idea that the pursuit of technological innovation (or, more broadly, scientific knowledge) has to be either guided by government action or by an appropriately designed institutional setting.

Recently, however, important contributions by Boldrin and Levine have tried

[1]The former favouring arguments like (positive) external economies of scale, incomplete appropriability, uncertainty and efficient dissemination; while the latter is usually based on partial equilibrium or game theoretical models of firm behaviour that try to determine important variables such as R&D investment intensities, introduction rates for new innovations, number of firms, or what market structure is more amenable to innovative activity.

to argue for a repeal of conventional wisdom. Using a broad range of tools, such as dynamic general equilibrium theoretical models and case studies, they have argued that socially efficient levels of innovation do occur under a perfectly competitive setting, with little to no intellectual property rights[2]. Concurrently, a pair of working papers by Quah ([20] and [21]) extend the breadth and scope of Boldrin and Levine by carefully analysing the role that infinite expansibility[3] and nonrivalry play in these models[4].

The purpose of this dissertation is to expand one of the possible formulations explored by Quah [20] by allowing consumers in that model a more active role, while maintaining fixed assumptions regarding the property rights structure implicit therein. We will study what implications this change will bring about on the issue of appropriability in the competitive outcome and whether the decentralised outcome will be able to support the social optimum.

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[2]For brevity's sake, throughout this document further references to these might be abbreviated to IPRs.

[3]Which has as its counterpart in the traditional economics of innovation the idea that ideas spread costlessly and as quickly as possible.

[4]In contrast with Romer [25], where nonrivalry is argued to be "inextricably linked to nonconvexities". These nonconvexities take on a different role in the articles discussed.

2. Literature Review

2.1 Early Forays

Of the initial contributions mentioned in the Introduction[5], the work of Nelson [16] focuses on the relationship between basic scientific research and inventive activity and the role of social institutions in providing sufficient quantities of the former. The argument is that the closer one gets to pure applied science, the smaller the uncertainty regarding the outcome of a given scientific experiment, i.e., the more likely it is to conduce to a useful invention. Hence, firms are more capable of appropriating the surplus generated by this sort of research. At the other end of the spectrum, however, basic research is so unpredictable and distanced from actual inventions that it seems unlikely that firms would engage in this sort of scientific enquiry, particularly because the benefits of which, though uncertain, would be widespread and the private firm would, likely, not be able to fully appropriate all of the value generated.

While it would be possible to include Nelson's distinction into current discussions of intellectual property rights and the production of innovations, the approach favoured here does not explicitly take into account any significant differences between basic scientific research and applied research. The formulations explored

[5] It should be noticed that other important works such as Shell [30] have made important early contributions. Neglect to explore them at length here does not detract from their importance, but the crux of the arguments is broadly encompassed in the literature mentioned.

herein relate to the production of intellectual assets or inventions that can be readily transformed into consumer goods or consumption flows. They are at one extreme of the basic/applied research spectrum and, hence, try to ascertain whether the benefits accrued to inventors in a competitive setting are sufficient to reward them for the costs incurred in research activity. It could be argued, however, that even though inventors are appropriately rewarded in a competitive, no IPRs world, major improvements in basic scientific knowledge (which are the purpose of basic scientific research) would reduce the costs of engaging in applied research and, hence, increase innovative effort at the applied end of the spectrum.

Remarkably, however, is that beyond the discussion of the social institutions required to provide adequate resources into basic research; the author seems to imply that firms, in order to be able to extract any value from their inventions, must patent this newly acquired knowledge.

> "In order to capture the value of the new knowledge in fields which the firm is unwilling to enter, the firm must patent the practical applications and sell or lease the patents to firms in the industries affected." [16]

In addition to this, Nelson seems to implicitly adhere to the Schumpeterian idea that a perfectly competitive setting is unlikely to produce great amounts of investment in innovation. Where in [16] "'multiple-product giants"' are the only firms society can rely on to produce basic research of their own accord, Schumpeter's [29] own view involves firms who engage in what was(is) commonly described as anti-competitive behaviour, featuring patents as a firm practice rather than a mechanism designed specifically with that intention:

> "Hence it becomes necessary to resort to such protecting devices as patents or temporary secrecy of processes or, in some cases, long-period contracts secured in advance." [29]

Finally, in what is probably the most influential article on the particular features of the transmission of information in a free enterprise economy, Arrow [1] outlines the basic case for market failure in the allocation of resources for invention. It's based on three essential features of information and its transmission: indivisibility, inappropriability and uncertainty. The first of these relates to the property of information or knowledge that, once it has been "created" in an appropriate sense, infinite copies can be readily made, theoretically, without detracting from the use of the original instance[6]. Inappropriability refers to the lack of sufficient property protection for the creator of the innovation, which is unable to reap the full benefits brought about by the introduction of the innovation. It is closely linked to the fact that indivisibility essentially means that because there is only an initial unit of an idea, there is zero marginal cost of reproduction and, hence, a zero price in a perfectly competitive market. Because the innovator sees little, if anything, of the gain brought about by his labour or the costs he incurred to finance it, the incentive to engage in such activities is either very small or non-existent - both of which are surely not optimal as the innovation does bring about some social benefits. Finally,

[6] Romer [24] sees this non-convexity as an example of nonrivalry. The use of nonrivalry in the context of this thesis, however, is closer to the interpretation given by Quah [20], as infinite expansibility is deemed to be a better approximation for what is meant as indivisibility.

markets fail to correctly allocate resources to invention because, as an inherently risky activity, entrepreneurs are not able to fully hedge their investments, which inevitably decreases investment so long as a majority of agents are risk averse. The lack of appropriate market instruments available to override this problem is to pool these resources in common enterprises, i.e., large firms, which are then capable, by running multiple projects at once, of dealing with such high levels of uncertainty.

However, despite seeming to corroborate Nelson's and Schumpeter's underlying hypothesis that some degree of market power or share is required for innovation to take place, the crux of Arrow's contribution is to compare the incentives to innovation present whether the firm is a monopolist or a price-taker under a given cost structure. He shows that in a competitive setting, a firm will have a higher incentive to invest in research & development, despite still coming short of what would be the case in the social optimum.

A common thread running throughout these works is that without some kind of legal protection for inventors, the costless transmission of information, coupled with nonrivalry, will inevitably drive the price of the invention to zero as soon as it is released by its creator; and the incentive to innovate will be so low that socially desirable levels of innovation will not be achieved.

2.2 Partial Equilibrium Models

Starting with Nordhaus [17], Scherer [27] and [28], and Kamien and Schwartz [11], [12] and [13] (among many other contributions), economists began to worry significantly more about other aspects of invention, such as the optimal time to implement an innovation and what kind of market structure would support a socially desirable outcome or the optimal duration time for a patent. Most of these readily pointed out several problems with a full fledged market system, which would lead to either belated or delayed introduction times, duplication of investment or even excess capacity.

Crucially, these contributions eventually converged to what are known as "patent race" models, where there is a steady and perpetual flow of rewards (guaranteed by patent protection or other media) to whoever obtains the first implementable copy of the process of product. Loury [15] argues that, under uncertainty, atomistic competition would yield an excess of innovation in comparison to the social optimum[7] as well as an excess of firms entering the "race" for the patent[8].

Other contributions in this field, such as [7], expand the analysis pioneered by Arrow and present solid arguments for industry concentration and higher investment in R&D not only to be highly correlated, but endogenously determined, though this may not necessarily mean reduced effective competition[9]. Excessive duplication in R&D are also prominent features of the models presented, results which validate previous research on the topic; while it does not seem at all evident that competitive markets provide insufficient resources to innovation - quite to the contrary, there

[7]This, as Loury points out further in the model, is simply a result of an excessively high reward. If patents are appropriately reduced, this need not be the case.

[8]Other results in his paper involve some amount of excess capacity by allowing increasing returns in the research technology in some initial range, but these were shown not to be general by Lee and Wilde [14].

[9]Not an entirely new idea, if we care to remember Schumpeter [29].

seems to be excess investment due to uncertainty regarding the first firm to gain access to it. In [8], Stiglitz and Dasgupta find similar results, extend some of these and eventually conclude that without some form of market power (of which IPRs are a special case), no firm engages in R&D activity. This leads to their argument that

> "(...) in those sectors of the economy where technological change is significant, the analysis of competitive market equilibrium within the framework provided by the traditional competitive equilibrium (e.g., Arrow-Debreu) model is of limited applicability: competition in R&D necessitates imperfect competition in product markets."

Building on the game-theoretic foundations proposed in Scherer [27], Loury [15] and Lee and Wilde [14], Reinganum [22] uses a dynamic game of research and development under various degrees of patent protection to show that increased appropriability leads to higher levels of investment under competition.

A useful survey of this literature can be found in Reinganum [23], where one will find that many of the results in this line of research establish that a competitive market structure cannot be a suitable framework in which to analyse issues pertaining to the activity of generating knowledge or innovations.

2.3 Other research

Works in the vein of the models described earlier do not exhaust the literature on the issue, however, and although recent contributions still use the framework of patent races to analyse issues of patent law design (e.g., [9]), other approaches have been proposed by various authors. One prominent example is that of Romer [25], which presents arguments for introducing nonconvexities, i.e., increasing returns, in firm's production functions. The endogenous growth literature stemming from these assumptions (relevant contributions include [26], [24] or [10]) has contributed significantly to the discussion by pointing out not only inadequacies in provision of goods within a competitive market, but also the implications it might have on economic growth. Research on sequential innovation by Bessen and Maskin [2] and Pollock [18] argues that while patent protection, under certain conditions, provides higher net social welfare; if innovation is sequential, protection is in fact more likely to induce sub-optimal levels of innovation. Finally, Boldrin and Levine [5] specify a model in which firms are allowed to choose between private rent-seeking (e.g., trade secrecy) and public rent-seeking (e.g., IPRs). They argue that IPR enforcement is only beneficial under certain parametrisations of the model, even when the government conceding the patents is a benevolent social planner seeking to maximise social welfare.

2.4 The ‘New’ Economics of Innovation

While the bulk of the available academic literature on the issue of production of knowledge[10][11] seems to have on good authority that innovation cannot occur in perfectly competitive markets cannot occur at levels that are socially desirable, with some authors even arguing that IPRs only partially solve the problem of appropriability and might generate other inefficiencies of their own.

In stark contrast with the lines of research described in the preceding section, Boldrin and Levine[12] have made several contributions to the case that intellectual property rights are, in general, not necessary for innovation to occur at socially desirable levels and create unnecessarily inefficient monopolies. However, a stark reminder of the sensitivity of these results to different specifications of the timeframe in which agents can make their choices has been put forth by Quah [20]. A detailed review of these studies will follow.

2.4.1 The Case Against Intellectual Monopoly

In a small introductory piece, Boldrin and Levine [3] set out the case not against property *per si*, but against the extension of the usual notions of property

[10]This term encompasses a wide variety of topics, ranging from information or knowledge production to investment in research and development. While there are significant differences between what various articles purport to study, they are sufficiently close to be included in this very broad category.

[11]It should be noted that the preceding review was perhaps purposely lopsided. While this is not entirely correct, it does seem to reflect the *statu quo ante* in economics prior to the “new” economics of innovation of Boldrin and Levine and Quah. A more comprehensive review can be found in Pollock [19].

[12]The expression in the title of the section originates from a summary page by Rufus Pollock on an early version of [6] and on [21]. It can be accessed here: `http://www.rufuspollock.org/economics/papers/boldrin_levine_quah.html`

into control over the use of the property once sold. They argue that the IPRs not only protect property (unless you sell your idea or divulge it in some other way, no one can access it), they allow the creator of idea, or whoever holds the patent, to retain control over the use to which the idea is put. This "downstream licensing" is viewed as having a negative impact on innovation in light of [6], in which competitive markets produce a socially efficient outcome.

A subsequent article [4] introduces the production of intellectual assets as a form of capital in a macroeconomic model[13], allowing for reproduction of capital of the current quality or investment in new qualities. In this work, they show that the competitive equilibrium is Pareto efficient, whereas intellectual monopoly, i.e., IPRs, is associated with less innovation in the aggregate, with an indivisibility in the production function worsening the case for patent protection[14].

2.4.2 Perfectly Competitive Innovation

The centrepiece of Boldrin and Levine's proposal of abolishing intellectual property rights, [6], this paper explores a wide variety of general equilibrium asset pricing models, in which ideas or innovation are the capital good and share prices at period 0 represent the return the creator receives from his inventive efforts. Key to this line of research is the notion that although ideas are assumed to be perfectly

[13]This formulation predates Quah [20] and can be seen as the theoretical foundation for the model to be developed in section 3, although the approach followed here is much simpler and more restrictive than the one followed in [4]. Pollock [19] is to be credited with noting the apparently obvious link between this formulation and that which is used for capital in macroeconomic models.

[14]The indivisibility here alluded does not correspond to a case where $f(0,k) = 0$, $f(1,k) = f(n,k)$, as pointed out by Pollock [19]. It amounts to requiring a certain fixed amount of the current quality of capital $\underline{k}$ in order to produce any amount of the new quality of the capital good.

nonrival[15], they are useless in themselves. For ideas to have economic value, they must be embedded in goods, processes or people. For this reason, they assume that the idea is embedded in the first copy of the good. This assumes away the problem of the indivisibility in the production function because the copying production used is simply a linear function of the stock of goods containing the initial idea.

Hence, the issue of the initial indivisibility/nonrivalry would appear to have been entirely dealt with by the specification used in the model. It turns out that is not the case. If the reproduction rate of the intellectual goods is infinite, then it can be argued that there is a second indivisibility, now in goods embodying ideas, i.e., inventions or innovations. Pollock [19] makes a compelling case that the indivisibility in ideas might be a problem if the production function can be expressed in terms of actual physical capital and ideas ($f(n,k)$, as alluded to above), which would then yield a second period price of 0 for the innovator. As for the second indivisibility, Boldrin and Levine show that, for a wide range of models, the return on the innovation is still bounded away from 0, even when allowing for immediate and infinite dissemination. It should be noted, however, that the authors take issue with the assumption of costless and boundless dissemination. Even in the case of ideas, while the presence of an indivisibility means that everyone could, theoretically, possess the idea instantaneously from the moment of its creation, the fact is that it is unlikely that such an outcome could be achieved. In fact, Boldrin and Levine

[15]In that their use or storage does not detract from other uses; which implies that the idea or blueprint to manufacture a certain product or provide some kind of service can be used by an arbitrarily large number of people without affecting the use of others of that same idea.

humorously point out "the irony of professors - whose salary derives entirely from the difficult and time consuming nature of communicating old ideas - arguing that ideas are freely and costlessly transmitted public goods".

Beyond the theoretical treatment of innovation pricing, the authors provide an interesting array of anecdotes which highlight cases in which production of goods or services involve generating new ideas; in contexts where traditional theory would argue that no protection would lead to innovation not occurring.

2.4.3 24/7 Competitive Innovation

Building on the framework developed by Boldrin and Levine, Quah [20] outlines a general model from first principles, extending at many points the analysis previously undertaken by those authors. The basic model assumes a copying rate γ for all of the instances of the good in its possession that are not being made available to consumption. The ones that are available can also be copied, albeit at a degraded rate $\hat{\gamma}$. There is rivalry in his baseline model in the sense that when a copy of the good is being used for consumption, it cannot be used in the copying process at the normal rate; instead entering the production function multiplied by the degraded rate. The property rights regime is similar to that prevailing in Boldrin and Levine [6], in which whoever purchases a full instance of the good (rather than a unit of consumption flow) is awarded full property rights. This means that although instances are allocated to consumption, consumers themselves do not reap any of the gains, as firms retain the product $\hat{\gamma} c_t$, where c_t is the number of copies allocated to consumption.

An extension of this model to a case where the good is nonrivalrous is presented in a following chapter and is shown to imply that instances comprising the entire stock of the good at time t can be copied at rate γ while those instances that are allocated to consumption are reproduced at rate σ. Evidently, both in social and competitive optima all of the available copies are consumed, which means they reproduce at rate γ and at rate σ. A final extension has consumers appropriating the proceeds of the copying from units allocated to consumption, with rivalry being introduced in the model. This approach scraps the consumption flow approach and essentially says that there is no difference between purchasing an instance for copying or consumption, because there is no intellectual property regime and consumers may do whatever they please with their copies.

The results in this paper can be summarised as follows: in the discrete time version of these models, Boldrin-Levine results are confirmed, with the return on the innovation being bounded away from zero even with infinite expansibility (in the sense of the copying rate approaching to infinity); in the continuous time version of the models, these results do not hold and the competitive equilibrium fails to deliver the social optimum when there is infinite expansibility. The interpretation for these results is straightforward. When time is discrete, agents can only act in finite periods of time, which means that the lag between the introduction of the innovation and further copying is sufficiently large to ensure a socially optimal outcome. In other words, innovators have enough "time" to reap gains from their endeavours and that ensures that the best possible allocation is achieved. If agents can react to the introduction of the innovation arbitrarily quickly[16], however, they essentially deprive the innovator of that initial period in which he can appropriate a sufficient amount of the social value created and hence "entry" in the sense of purchasing copies and selling them dissipates his rents entirely within an instant.

Another contribution by Quah [21] generalises the approach followed in the previous paper in a considerably more complex way by allowing a variety of ideas to be instantiated at period 0. His results show that if the indivisibility in the instantiation implies instantiation to a fixed quantity, the decentralised equilibrium fails to produce an optimal outcome. If, on the other hand, if instantiation can be in any amount (though bounded), an "invisible hand" theorem establishes that a competitive innovation equilibrium maximises social welfare.

[16]This is the crux of the paper, and the reason for which it is titled "24/7 Competitive Innovation".

3. The Model

This proposal focuses on the technological process describing the law of motion for knowledge products (a good or process that embodies an innovation or innovative combination of previous goods) developed in section 3 of Quah [20]. The assumption there mimics that of Boldrin and Levine [6], in that copying rates are not infinite over a given period of time, which implies that, as soon as an initial copy of the good has been obtained, an endless number of copies cannot be created instantly. It also assumes that there are no property rights in what concerns the physical copies of the knowledge product, which can be readily purchased by firms entering the market to compete with the initial holder of the good.

Implicitly assumed in this formulation is that, while purchasing full copies allows firms to enter the market directly, consumers only have access to the 'consumption stream', in that they are allowed to use copies of the good for consumption purposes, but they do not own those copies and are not able to copy them through any means. Finally, consumption and savings are thought to be rivalrous, so that when instances or copies of the knowledge good are allocated to consumption, they cannot be used in normal copying by the firm.

Notation follows that in Quah [20]: s_0 is the initial amount of the innovative good that is present at the beginning of time 0, s_t is the stock of the good at time t, c_t is the amount of the good that is allocated to consumption services at time t and

γ is the copying rate which transforms an s_t amount of the good into an $s_{t+1} = \gamma s_t$ at period $t+1$. The use of $\hat{\gamma}$, the degraded copying rate in [20], is discarded here, as this quantity now depends on consumer effort and takes the form $g(e_t)$.

3.1 Copying Technology

Even though copying by consumers is not allowed in this specific setting[17], firms themselves, because they remain the owners of the good, are able to copy it at a degraded rate. In Quah's [20] version, this degraded rate is a given and neither firms nor consumers have any effect in the rate at which consumption allocated instances can be copied.

This formulation, however, assumes that consumer effort has an impact in this copying rate, either because consumers can choose different ways to access the knowledge good or because its use requires some previous knowledge (more or less like using new software). A possible justification for this technological setting is presented in [20]:

> To interpret the specification, consider a hypothetical copying technology for videogames. After the first few instances of the game exist, all necessary resources can be devoted to creating further copies of it. Copying proceeds at rate γ. Alternatively, the firm can set aside some instances of the videogame for employee or costumer play. But then the bit-manipulating copying machines that are working off those copies

[17]This can be thought as there being some physical properties of the goods which prevent consumers from using them for copying purposes.

slow down to the degraded copying rate $\hat{\gamma}$. This might happen either because the magnetic heads reading the hard disk medium have to allow for conflicting demands, or because the firm's employees are distracted and cannot operate at maximum efficiency.

Introducing effort simply means that consumers retain some control over how they consume, even if firms retain full control of the copies. Hence, future copies of the good depend on present copies that are not allocated to consumption, which are copied at rate γ, and instances that are currently being consumed, which are reproduced at rate $g(e_t)$. An interpretation for this closely parallels Quah (2002). If we assume that firms have to provide some costumer assistance to less knowledgeable users, it allocates some of its resources to that task, thereby reducing the resources (either human or technological) available to create more copies of the good.

$$\forall \text{ integer } t \geq 0 : \; s_{t+1} = \gamma(s_t - c_t) + g(e_t)c_t = \gamma s_t - [\gamma - g(e_t)]c_t \tag{3.1}$$

Where:

$$\lim_{s>0} g'(e) = \infty \tag{3.2}$$
$$g'(0) > 0$$

The tension between consumption and savings stems from the rivalry between the two in what concerns the use of the good, as firms only reap benefits if they can sell their product, while depending on saving some copies so that they will have instances to sell in future periods.

As in the original formulation, we could allow $g(e_t) = 0$, i.e., regardless of the representative consumer's choice of effort, copies allocated to consumption could not be copied. This case would closely mimic the production of goods like music CDs or software, since when the consumers purchase their copy, they are not legally allowed to copy it and compete with the original seller. This is a particular case which, however, is not of great interest to this analysis because we want to investigate what effect consumer effort will have on the copying rate of instances of the good being consumed.

A final assumption is that even if consumers exert no effort, firms are still able to obtain copies from these instances, albeit at a significantly degraded rate. This implies setting $g(0) > 0$.

3.2 Consumer Preferences

Consumers derive utility from consumption of the knowledge good and dislike exerting effort, which leads to the following preference structure:

$$V = \sum_{t=0}^{\infty} \beta^t [U(c_t) - f(e_t)] \tag{3.3}$$

The utility from consumption component is assumed to be concave, monotonically increasing and at least twice continuously differentiable, with the following Inada condition holding:

$$\lim_{c \to 0} U'(c) = \infty$$

As for the disutility from effort component, $f(e)$ is assumed to be convex (with $f'(\cdot)$ and $f''(\cdot)$ both positive) and $f(0) = 0$. When no effort is exerted, $W(\cdot)$ simply becomes

$$W = \sum_{t=0}^{\infty} \beta^t U(c_t)$$

3.3 Socially Efficient Outcome

Social efficiency is defined to be a feasible outcome, i.e., one that respects the resource constraint, which maximises the function in (3.3) given an initial amount of the knowledge product s_0.

We can now define the value function $V^*(\cdot)$ as the function in (3.3) evaluated at the constrained optimum:

Definition 1 *Given s_0. a socially efficient outcome maximises (3.3) subject to the possibility set C implicitly defined in (3.1)*

$$V^*(s_0) \equiv \max_{\{c_t \in C, e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [U(c_t) - f(e_t)]$$

3.3.1 Equilibrium in Two Periods

In a two period setting, this economy ends after period 1, so that the resource constraint only relates quantities between period 0 and period 1, because there is nothing after the end of the world and s_2 is defined to be zero.

Hence, the social planner faces the problem of maximising consumer preferences, given an initial amount of the knowledge good[18]

The social planner's problem is the following

$$\max_{c_0,c_1,e_0,e_1,s_1} \; U(c_0) - f(e_0) + \beta[U(c_1) - f(e_1)]$$

$$\text{s.t.} \qquad c_t \leq s_t, \; t = 0, 1$$

$$s_1 = \gamma s_0 - [\gamma - g(e_0)]c_0$$

The following also holds:

Condition $\mathfrak{A}$: The copying rate γ is bounded from below[19]

$$\gamma > g(e_0^*) + \beta^{-1} \times \frac{U'(s_0)}{U'(g(e_0^*)s_0)}$$

This restriction implies that the firm's copying rate has to be higher than the degraded rate plus a parameter that reflects the ratio of marginal utilities between the two periods. The intuition provided in [20] can be applied virtually without

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[18] Strictly speaking, innovation is assumed to have taken place. As in [6] and [20], costly instantiation, i.e., the costly act of creating the first reproducible good from an idea, has already taken place.

[19] This condition closely parallels the condition presented in page 15 of [20].

change to this case.

If the copying rate of consumption allocated instances of the good were allowed to take the value 0, this condition would hold trivially, as the ratio of marginal utilities would converge to 0 and γ would need only be larger than 0 for the condition to hold.

As $g(e)$ grows larger, however, the condition will be increasingly tightened, since both the fist part half and the second half of the right hand side of the equation will become larger. Thus, as $g(e)$ becomes larger, so must the difference between γ and $g(e)$.

Using a utility function of the CRRA family such as

$$U(c) = \begin{cases} \frac{c^{1-\theta}}{1-\theta}, & \text{for} \quad \theta > 0 \quad \text{and} \quad \theta \neq 1 \\ \log c, & \text{for} \quad \theta = 1 \end{cases}$$

the condition becomes

$$\gamma > g(e_0^*) + \beta^{-1}[g(e_0^*)]^\theta$$

Thus, the difference between the normal and degraded copying rates must be at least $\beta^{-1}[g(e_0^*)]^\theta$ if condition $\mathfrak{A}$ is to hold.

The socially optimal outcome is the solution to the social planner's problem presented earlier and it is characterised by the following proposition[20].

[20] All proofs can be found in the technical appendix.

Proposition 1 *If condition $\mathfrak{A}$ holds, then the following holds at socially efficient allocation*

$$c_0^* > 0$$

$$c_1^* = s_1^* = \gamma s_0 - [\gamma - g(e_0^*)]c_0^*$$

$$e_0^* > 0$$

$$e_1^* = 0$$

$$U'(c_0^*) = \beta U'(c_1^*)[\gamma - g(e_0^*)]$$

These results are quite intuitive. The first two give us the optimal values for consumption and it is easy to see that in period 1 the optimal choice is to consume every copy because it does not make any sense to save up to a non-existing future. The same reasoning applies to the effort level in the same period: since there is no benefit in exerting effort (the benefits of doing so in one period are felt only in the following one), the optimal choice is to exert no effort and simply consume everything.

As for the equation relating marginal utilities, it simply reflects the representative consumer's subjective valuation of future periods relative to the present, as well as the relative benefits of foregoing consumption today in exchange of a higher level of consumption tomorrow. An increase in either the discount factor or the difference between the copying rates will make either future consumption more valuable (increase in β) or make more copies available by then (increase in $[\gamma - g(e_t)]$).

Consumers will optimally respond to this imbalance by foregoing consumption at a present date in favour of more future consumption.

3.4 Decentralised Equilibrium

In a competitive market, firms supply a fraction of their current stock of the knowledge product as a consumption flow, sold at price p_t, while keeping the remaining amount of s_t as an asset which can be copied by the company or sold at price q_t. Since it is immaterial whether the consumer chooses to purchase it as a share in the company or buy a full copy in order to set up his own firm[21], describing s as an asset means that it can either be interpreted as stock in the firm or as purchasing a good which embodies the idea and its own "recipe"[22].

There are quite a few implicit assumptions in this characterisation of the market. First of all, the assumption of no IPRs means that when the asset or product is purchased, anyone can set up his or her own company and compete in an unfettered market. This is quite different from purchasing the consumption flow, which does not allow consumers to retain any control over the good itself (excepting, as assumed, the rate at which the firms are able to use the good for copying purposes).

An example of this is approximated by DVD rental under some existing legal regimes. Everyone is entitled to purchase a copy of the DVD (and paying a

[21]This is due to the linearity in the production technology. Although the $g(e)$ parameter is non-linear in effort, it is fully characterised by the of level effort. Since effort will depend only on the initial parameters of the model (γ, β and the intertemporal elasticity of consumption), it can be interpreted as an exogenous parameter.

[22]Recipe here simply means the technical knowledge required for obtaining more copies of the good.

differentiated price) and set up their own DVD rental firm (although they are not allowed to obtain further copies from that initial unit) and compete with others in the market. Consumers, on the other hand, pay a smaller amount (in our model approximated as p_t) and are allowed to enjoy the flow of consumption that, in this case, corresponds to viewing the movie. If we set the rental period to 1, this is simply the case when $g(e) = 1$[23]. One other way of interpreting our assumption regarding consumer effort is by considering that consumers would be able to return the DVD earlier, so that in the unit period, the firm would be able to obtain some copies from that single unit.

Since consumers are assumed not to have access to any kind of copying technology (certainly a strong assumption), this formulation sits somewhere between the current legal regime where creators, legally, have full appropriability, i.e., not even DVD rental stores are allowed to obtain their own copies and a regime in which there is no legal restriction and both consumers and firms are allowed to copy and enter the market as suppliers. Thus, this model simply amounts to relaxing legal requirements slightly in favour of consumers and checking whether social efficiency may be obtained in a decentralised market outcome.

The representative consumer's budget constraint takes the form

$$p_t c_t + q_{t+1} s_{t+1} \leq q_t s_t \tag{3.4}$$

[23] Firms are not allowed to access the DVD during that period and only regain the exact amount that was originally rented out.

While firms maximise value while being bound by the technological constraint in (3.1)

$$q_t s_t \equiv \sum_{j=0}^{\infty} p_{t+j} c_{t+j} \tag{3.5}$$

Therefore, a decentralised market outcome is defined as follows.

Definition 2 *Given an initial stock of the knowledge product, s_0, a competitive equilibrium is characterised by the following sequences*

$$\{p_t^{m*}, q_t^{m*}, c_t^{m*}, e_t^{m*}, s_{t+1}^{m*}\}_{t=0}^{\infty}$$

with both consumers and firms taking prices p and q as given, while consumers maximise (3.3) subject to (3.4) and firms maximise value (3.5) subject to the technological constraint in (3.1).

As in [20], linearity in the copying technology means that the "competitive equilibrium will be independent of the ownership of s_t" (Quah, [20]). The equilibrium price of q_0 will, hence, determine whether innovation takes place or not, as it corresponds to the present value of the initial instance of the idea or innovation.

3.4.1 Equilibrium in Two Periods

Similarly to the socially efficient case, we examine the case where the economy lasts for only two periods.

The representative consumer's problem takes the form

$$\max_{c_0,c_1,e_0,e_1,s_1} \; U(c_0) - f(e_0) + \beta[U(c_1) - f(e_1)]$$

$$\text{s.t.} \quad q_1 s_1 + p_0 c_0 \leq q_0 s_0$$

$$p_1 c_1 \leq q_1 s_1$$

While firms face

$$\max_{c_0,c_1,s_1} \; q_0 s_0 = p_0 c_0 + p_1 c_1$$

$$\text{s.t.} \quad c_t \leq s_t, \; t = 0, 1$$

$$s_1 = \gamma s_0 - [\gamma - g(e_0)] c_0$$

Provided condition $\mathfrak{A}$ holds, the following is true

Proposition 2 *The competitive allocation, denoted by*

$$\{c_0^{m*}, c_1^{m*}, s_1^{m*}\} \quad \text{with} \quad e_0^{m*} = e_1^{*} = 0$$

is not identical to the socially efficient allocation

$$\{c_0^{*}, c_1^{*}, s_1^{*}\} \quad \text{with} \quad e_1^{m*} = 0 < e_0^{m*}$$

while market clearing quantities are achieved at prices

$$p_0^{m*} = U'(c_0^{m*})\lambda_1^{-1}$$

$$p_1^{m*} = p_0^{m*}[\gamma - g(0)]^{-1}$$

$$q_0^{m*} = \gamma p_1^{m*}$$

$$q_1^{m*} = p_1^{m*}$$

Proposition 3 *At the decentralised equilibrium, socially efficient quantities are not affordable, i.e.*

$$p_0^{m*}c_0^* + p_1^{m*}c_1^* > s_0 q_0^{m*} = p_0^{m*}c_0^{m*} + p_1^{m*}c_1^{m*}$$

All proofs can be found in the technical appendix. Because socially efficient quantities are not affordable in the decentralised equilibrium (contra [20]), the procedure followed in the appendix consists of finding market clearing quantities and the price structure that supports them, rather than using the approach followed in [20]; which simply takes the socially efficient allocation and looks for the price structure that would support it in the competitive equilibrium.

That markets fail to efficiently allocate resources in this setting is entirely due to the representative consumer's choice not to exert effort in any period. Because of excess appropriability, consumers have no direct benefit for engaging in costly activities and, hence, choose not to undertake efforts which would improve the resource constraint and make the socially efficient outcome affordable.

In what concerns the broader argument over the necessity of IPRs as welfare enhancing mechanisms, these results show that, quite to the contrary, when there is excess appropriability by firms, a socially efficient outcome is not feasible because consumers have no incentive to contribute, through their efforts, to improving the rate of acquisition of new instances of the good.

Checking whether q_0 has a "positive valuation" in the decentralised outcome is now required to establish that, in equilibrium, there is a positive return on innovation. Following [20], we let $\lambda_1 = 1$ so that marginal utility in period 0 becomes our unit of measure. Applying this to the equation for q_0^{m*}, we get

$$q_0^{m*} = \gamma U'(c_0^{m*})[\gamma - g(0)]^{-1} \geq U'(c_0^{m*}) > 0$$

Even as $\gamma \rightarrow \infty$, $q_0^{m*} > 0$. From the discussion in [20], the role on infinite expansibility is recognised to be of central importance in the results of this family of models. This formulation maintains that basic result, which was also proposed in [6] and [3]: by taking the limit as γ approaches infinity, we see that q_0^{m*} is still positive.

A further implication of the model is that q_0^{m*} is not only positive, but also equal to the "social value of a marginal increment in the initial asset s_0" (Quah, [20]). This is established in the following proposition.

Proposition 4 *In the decentralised equilibrium,* $q_0^{m*} = s_0^{-1}(p_0^{m*}c_0^{m*}+p_1^{m*}c_1^{m*})$*, where,* *assuming that* $\lambda_1 = 1$, $q_0^{m*} = V'(s_0)$ *follows.*

Despite the fact that the market outcome fails to properly allocate resources, simply by assuming that consumers have some say in what the copying rate of instances allocated to consumption will be, these results are not wholly disparate from those in [6] or [20]. In fact, even though social efficiency is not attained by the market outcome, the price of the asset s_0 at time 0 is still positive. This implies that all innovations costing up to q_0^{m*} will be undertaken, whereas more expensive projects are not pursued in the competitive equilibrium.

Although the decentralised outcome still allows for innovation to occur (provided their cost does not exceed q_0^{m*}), it is not clear whether the social optimum is more conducive to innovative activity. By checking what price structure would support the socially efficient allocation, it is easy to check that the required price for period 1 can be higher, lower or even equal to the competitive price. This, in turn, implies that the return on the asset, q_0, required for the socially efficient allocation can also be higher, lower or equal to that of the competitive case. Thus, it is not clear whether the decentralised equilibrium is more, less or equally conducive to innovative activity, as it will depend on optimal consumer choices for c_1, optimal supplies and the price structure supporting these quantities.

Proposition 5 *The pricing structure that supports socially efficient quantities is given by*

$$p_1^* = p_0^*[\gamma - g(e_0^*)]$$

where $p_0^* = U'(c_0^*)$ *(by setting* $\lambda_1 = 1$*). This implies that*

$$p_1^* = q_1^* \lesseqqgtr p_1^{m*} = q_1^{m*}$$

$$q_0^* \lesseqqgtr q_0^{m*}$$

Whereas in the price structure supporting the decentralised outcome the price of a unit of consumption flow decreased at rate $[\gamma - g(0)]$ from period 0 to period 1, in this hypothetical pricing scheme the rate of decrease is slower. All things equal, this would imply a higher price in period 1 and a subsequent higher value of q_0. However, since consumption in period 0 is necessarily lower in the decentralised outcome, this implies that a hypothetical p_0 supporting the socially efficient outcome would be lower than p_0^{m*}. Hence, while one of the determinants of p_1^* decreases, the other increases and hence the total impact cannot be established. In turn, since one of the determinants of q_0^* cannot be determined, the total effect on the return on the asset is ambiguous and cannot be pinpointed without imposing further assumptions.

4. Discussion

The analysis undertaken in the previous chapter shows that under the legal and technological structure of Boldrin and Levine [6], when the copying rate of the goods allocated to consumption depends on consumer effort, the decentralised equilibrium fails to deliver the social optimum. Despite this, it should be restated that the implications for the return on innovation cannot be determined without further enquiries into the welfare trade-offs involved. It does, however, maintain that, in a competitive equilibrium, return on innovation is positive, even when the copying rate is allowed to approach infinity, even though it does not specify whether the return on innovation is more, less or exactly what it would be in the social optimum.

An implication of the formulation chosen is that the level of innovation induced by the competitive price sequence can be larger, smaller or exactly that which would prevail in the social optimum. What is unambiguous, however, is that the consumption sequence supported by the competitive price system is not optimal, so that even if there was to be an "optimal" level of innovation (in the sense of initial supplies of s_0), the time path of consumption would still be determined by a less than optimal amount of effort, implying that consumption choices in themselves would be sub-optimal.

An important result of Boldrin and Levine [6], that appropriability is not a concern under competitive pricing because the creators of the good are sufficiently rewarded in equilibrium, is confirmed by the results discussed in the previous chapter. In fact, excess of appropriation by firm is the cause of market failure, as consumers are not rewarded for their effort and, hence, choose not to exert any.

It could be argued that companies, knowing of this, would attempt to induce optimal consumer effort levels by reducing the options at the consumer's disposal. This could be, for example, not allowing different levels of access to the consumption flow[24]. In many cases, however, it might not be possible to do so. If we imagine a DVD rental service, unless the firm forces its consumers to watch the movie and return it immediately, they are allowed some discretion and, in all likelihood, delivery of the DVD would be delayed until the last possible instant. The key result of introducing this assumption is that, if consumers have any degree of discretion in the effort they exert in the consumption of some good, and if this has an impact on the firm's copying rate of these instances, then a decentralised equilibrium is not optimal.

Another argument is that firms could induce consumers into exerting a desirable amount of effort by offering discounts or other amenities. Although this could be useful in the case of DVDs - through offering discounts for speedy delivery, for example -, in cases where the copying rate depends on the level of user knowledge of the consumer (which in turn depends on effort exerted to acquire the knowledge), it is hard to see how the firm would be able to devise an incentive scheme that would

[24]The example of a streaming album or movie comes to mind easily: if the consumer is allowed to choose between "low bandwidth" or "high bandwidth", all consumers would choose "high bandwidth", potentially causing the company's servers to crash due to overload. A simple way of solving this is allowing only one option, which has consumers exert the amount of "effort" that would best suit the company.

induce optimal levels of effort.

An obvious extension of the argument made throughout this work would be to have consumers retain control over the copies they purchase, as in [20], but having the reproduction rate depend on the level of effort necessary to "operate" the copying technology.

Not so obvious extensions would necessarily have to address the remaining issues raised as far back as Arrow's seminal contribution. While the work of Boldrin-Levine and Quah attempts to address the issues of indivisibility and inappropriability - by restating what is meant by indivisibility and showing that appropriability need not be an issue under certain circumstances -, it entirely neglects the issue of uncertainty[25], which clearly plays a central role in the actual process of private and public allocation of resources for invention. Another criticism of this literature is the one presented by Pollock [19] in his literature review, concerning the true nature of the indivisibility and how it should be addressed if the process of innovation is to be realistically captured.

In short, while the formulation presented here and in [4], [6], [20] and [21] can and should be expanded to address these issues, the insights provided by this literature and the challenges they pose to conventional wisdom are sufficiently sound to be taken seriously and built upon.

[25]This is true for the general equilibrium models studied. Boldrin and Levine [5] do allow for uncertainty in the timing of the delivery, much in the vein of the patent race models described in the literature review.

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APPENDIX A

Technical Appendix

This appendix contains proofs to all the propositions presented in chapter 3 of the thesis.

A.1 Socially Efficient Outcome in Two Periods

Proofs of propositions 1 and 2 closely follow the Kuhn-Tucker formulation as outlined in [31].

Proof of Proposition 1 *The Lagrangean function for this problem can be written as:*

$$\mathfrak{L} = U(c_0) - f(e_0) + \lambda_1\{\gamma s_0 - [\gamma - g(e_0)]c_0 - s_1\} + \lambda_2(s_0 - c_0)$$
$$+\beta[U(c_1) - f(e_1) + \lambda_3(s_1 - c_1)]$$

The first order conditions imply:

$$
\begin{array}{llll}
c_0: & U'(c_0) - \lambda_1[\gamma - g(e_0)] - \lambda_2 = 0 & e_1: & \beta f'(e_1) \geq 0;\ e_1 \geq 0 \\
c_1: & U'(c_1) - \lambda_3 = 0 & \lambda_1: & s_1 = \gamma s_0 - [\gamma - g(e_0)] \\
s_1: & \lambda_1 = \beta\lambda_3 & \lambda_2: & s_0 \geq c_0;\ \lambda_2 \geq 0 \\
e_0: & \lambda_1 c_0 g'(e_0) = f'(e_0) & \lambda_3: & s_1 \geq c_1;\ \lambda_3 \geq 0
\end{array}
$$

From the FOC for e_1, we have that:

$$\beta f'(e_1) \geq 0;\ e_1 \geq 0$$

Clearly, if $e_1 > 0$, then $\beta f'(e_1) = 0$, implying that $f'(e_1) = 0$, which, under the assumptions made, doesn't happen at any point where $e_1 > 0$. Hence, $e_1^* = 0$. Candidate solutions can now be considered. One of these involves:

$$s_1^h = c_1^h = \gamma s_0 - [\gamma - g(e_0^h)]c_0^h \Rightarrow \lambda_3 \geq 0$$

$$s_0 = c_0^h \Rightarrow \lambda_2 \geq 0$$

The first order conditions imply:

$$\lambda_1 g'(e_0^*)c_0^* = f'(e_0^*)$$

$$e_1^* = 0$$

$$\beta\lambda_3 = \lambda_1$$

$$U'(c_0^*) = [\gamma - g(e_0^*)]\lambda_1 + \lambda_2 \Rightarrow U'(c_0^*) = [\gamma - g(e_0^*)]\beta U'(c_1^*) + \lambda_2$$

From the candidate solution, $s_0 = c_0^h$*. Hence:*

$$U'(c_0^h) \geq [\gamma - g(e_0^h)]\beta U'(c_1^h) \Rightarrow \gamma \leq g(e_0^h) + \beta^{-1}\frac{U'(c_0^h = s_0)}{U'[c_1^h = s_h^1 = g(e_0^h)s_0]}$$

Which violates condition $\mathfrak{A}$*. Therefore,* $0 < c_0^* < s_0$*.*

Another possibility involves $c_1^* < s_1^*$*. It is easy to check that the first order conditions would then imply:*

$$\lambda_3 = 0 \Rightarrow U'(c_1^h) = 0$$

Which clearly cannot hold at the optimum. Therefore, the socially efficient outcome *involves:*

$$0 < c_0^* < s_0$$

$$c_1^* = s_1^* = \gamma s_0 - [\gamma - g(e_0^*)]c_0^*$$

$$e_0^* > 0$$

$$e_1^* = 0$$

A.2 Competitive Equilibrium in Two Periods

Proof of Proposition 2 *The Lagrangean function for the representative firm's maximisation problem can be written as:*

$$\mathfrak{L} = p_0 c_0 + p_1 c_1 + \lambda_1(s_0 - c_0) + \lambda_2(s_1 - c_1) + \lambda_3\{\gamma s_0 - [\gamma - g(e_0)]c_0 - s_1\}$$

The first order conditions for this problem yield:

$$c_0 : p_0 - \lambda_1 - \lambda_3[\gamma - g(e_0)] \leq 0;\ c_0 \geq 0 \qquad \lambda_1 : \quad s_0 - c_0 \geq 0;\ \lambda_1 \geq 0$$

$$c_1 : p_1 - \lambda_2 \leq 0;\ c_1 \geq 0 \qquad \lambda_2 : \quad s_1 - c_1 \geq 0;\ \lambda_2 \geq 0$$

$$s_1 : \lambda_2 = \lambda_3 \qquad \lambda_3 : \quad s_1 = \gamma s_0 - [\gamma - g(e_0)]c_0$$

A possible solution involves $c_1^h < s_1^h$*. From the first order conditions,* $p_1 < \lambda_2$ *and* $\lambda_2 = 0$*. Since* $\lambda_2 = \lambda_3$*,* $\lambda_3 = 0$*, which is impossible because the resource constraint holds with equality - i.e., there are no other possible uses for the good, either it is* *saved and kept at the firm for copying or allocated to consumption. Hence,*

$$c_1^{m*} = s_1^{m*} = \gamma s_0 - [\gamma - g(e_0^{m*})]c_0^{m*}$$

There are three distinct possibilities for the optimal supply of c_0. *One of these is that* $c_0^h = 0$. *From the first order conditions, this implies:*

$$p_0 \leq \lambda_3[\gamma - g(e_0)] \Rightarrow p_0 \leq p_1[\gamma - g(e_0)]$$

If $c_0^h = s_0$*:*

$$p_0 = \lambda_1 + \lambda_3[\gamma - g(e_0)] = \lambda_1 + p_1[\gamma - g(e_0)] \Rightarrow p_0 \geq p_1[\gamma - g(e_0)]$$

If $0 < c_0^h < s_0$

$$p_0 = p_1[\gamma - g(e_0)]$$

Hence, the firm's optimal supply of c_0 *depends on what the market price of one unit of consumption flow at period* 0 *turns out to be. In short:*

$$\begin{cases} p_0 > p_1[\gamma - g(e_0)] \Rightarrow c_0^{m*} = s_0 \\ p_0 = p_1[\gamma - g(e_0)] \Rightarrow 0 < c_0^{m*} < s_0 \\ p_0 < p_1[\gamma - g(e_0)] \Rightarrow c_0^{m*} = 0 \end{cases}$$

In order to fully pin down the equilibrium quantities, the representative consumer's choices must be analysed. The Lagrangean function for the representative consumer can be written as:

$$\mathfrak{L} = U(c_0) - f(e_0) + \lambda_1(q_0 s_0 - q_1 s_1 - p_0 c_0) + \beta[U(c_1) - f(e_1) + \lambda_2(q_1 s_1 - p_1 c_1)]$$

The first order conditions for this problem are:

$$\begin{array}{ll} c_0 : U'(c_0) - p_0\lambda_1 = 0 & s_1 : \quad \beta\lambda_2 = \lambda_1 \\ c_1 : U'(c_1) - p_1\lambda_2 = 0 & \lambda_1 : \quad q_0 s_0 = q_1 s_1 + p_0 c_0 \\ e_0 : -f'(e_0) \leq 0;\ e_0 \geq 0 & \lambda_2 : \quad q_1 s_1 = p_1 c_1 \\ e_1 : \beta[-f'(e_1)] \leq 0;\ e_1 \geq 0 & \end{array}$$

The Inada condition rules out $c_0 = 0$*, which means that either* $c_0 = s_0$ *or* $c_0 < s_0$ *at the optimum. From the firm's first order conditions,*

$$c_0 = s_0 \Rightarrow p_0 > p_1[\gamma - g(e_0)]$$

Since the consumer's first order conditions imply:

$$\frac{U'(c_0)}{\beta U'(c_1)} = \frac{p_0}{p_1}$$

It is clear that (using the fact that $c_0^h = s_0$ *and* $c_1^{m*} = s_1^{m*}$*):*

$$\frac{U'(s_0)}{\beta U'(g(e_0^{m*})s_0)} > \gamma - g(e_0^{m*}) \Rightarrow \gamma < g(e_0^{m*}) + \frac{U'(s_0)}{\beta U'(g(e_0^{m*})s_0)}$$

Which violates condition $\mathfrak{A}$. *As for the level of effort, we need only notice that since* $f'(e) > 0, \quad \forall \quad e \in [0, \infty)$, $e_0^{m*} = e_1^{m*} = 0$. *Hence, in the decentralised equilibrium, the level of effort exerted in period* 0 *is* 0*, which implies a lower rate of copying for copies allocated to consumption than in the socially efficient outcome.*

Thus, market clearing quantities are given by $0 < c_0^{m*} < s_0$ *and* $c_1^{m*} = s_1^{m*} = \gamma s_0 - [\gamma - g(0)]c_0^{m*}$*, while prices are (from the first order conditions for the representative consumer):*

$$p_0^{m*} = U'(c_0^{m*})\lambda_1^{-1}$$

$$p_1^{m*} = p_0^{m*}[\gamma - g(0)]^{-1}$$

The prices for the underlying asset s, q_0 *and* q_1*, can be easily determined. Evidently, if* $c_1^{m*} = s_1^{m*}$, $p_1^{m*} = q_1^{m*}$. *As for* q_0^{m*},

$$q_0^{m*}s_0 = p_0^{m*}c_0^{m*} + p_1^{m*}c_1^{m*} = p_0^{m*}\{c_0^{m*} + c_1^{m*}[\gamma - g(0)]^{-1}\} = \gamma s_0 p_1^{m*}$$

which implies that $q_0^{m*} = \gamma p_1^{m*}$.

Proof of Proposition 3 *Since* $q_1^{m*} = p_1^{m*}$ *and* $c_1^{m*} = s_1^{m*}$*, as it easy to check from the representative consumer's first order conditions, total expenditure (or firm value)* *under the socially efficient outcome would be:*

$$p_0^{m*}c_0^* + p_1^{m*}c_1^* = p_0^{m*}c_0^* + p_0^{m*}[\gamma - g(0)]^{-1}c_1^* = p_1^{m*}\{c_0^*[\gamma - g(0)] + c_1^*\}$$

Now,

$$\gamma s_0 = c_1^* + c_0^*[\gamma - g(e_0^*)] < c_1^* + c_0^*[\gamma - g(0)]$$

Hence,

$$p_1^{m*}\{c_0^*[\gamma - g(0)] + c_1^*\} > p_1^{m*}\gamma s_0 = s_0 q_0^{m*}$$

Implying that the socially efficient outcome is not affordable at the decentralised equilibrium.

Proof of Proposition 4 *That* $q_0^{m*} = s_0^{-1}(p_0^{m*}c_0^{m*} + p_1^{m*}c_1^{m*})$ *was established at the end of the proof of proposition 2. As for* q_0^{m*} *corresponding to the social value of an increment in* s_0*, the proof involves taking the total derivative of* $s_0 = [\gamma - g(0)]\gamma^{-1}c_0^{m*} + \gamma^{-1}c_1^{m*}$ *in order to* s_0*, which yields*

$$\frac{ds_0}{ds_0} = [\gamma - g(0)]\gamma^{-1}\frac{dc_0^{m*}}{ds_0} + \gamma^{-1}\frac{dc_1^{m*}}{ds_0} \Rightarrow$$
$$\Rightarrow \gamma[\gamma - g(0)]^{-1} = \frac{dc_0^{m*}}{ds_0} + \frac{dc_1^{m*}}{ds_0}[\gamma - g(0)]^{-1}$$

Evaluated at the decentralised outcome, the value function simply becomes

$$V^{m*}(s_0) = U(c_0^{m*}) + \beta U(c_1^{m*})$$

Therefore,

$$\frac{dV^{m*}(s_0)}{ds_0} = U'(c_0^{m*})\frac{dc_0^{m*}}{ds_0} + \beta U'(c_1^{m*})\frac{dc_1^{m*}}{ds_0}$$

From the equation relating marginal utilities of both periods,

$$\beta U'(c_1^{m*}) = \frac{U'(c_0^{m*})}{\gamma - g(0)}$$

These combined give

$$\frac{dV^{m*}(s_0)}{ds_0} = \left\{\frac{dc_0^{m*}}{ds_0} + \frac{dc_1^{m*}}{ds_0}[\gamma - g(0)]^{-1}\right\}U'(c_0^{m*}) = \gamma[\gamma - g(0)]^{-1}U'(c_0^{m*}) = \lambda_1 q_0^{m*}$$

Setting $\lambda_1 = 1$*, we get:*

$$\frac{dV^{m*}(s_0)}{ds_0} = q_0^{m*}$$

Simply put, q_0^{m*} *corresponds to the social value of a marginal increase in* s_0*.*

Proof of Proposition 5 *This proof involves showing that a* q_0 *supporting the socially efficient outcome is given by* $p_0^h\gamma[\gamma - g(e_0^*)]^{-1}$*. It involves showing that if the previous equality holds, then the socially efficient allocation must be affordable. By* *assumption, the value of the firm is then given by*

$$q_0^h s_0 = p_0^h \gamma[\gamma - g(e_0^*)]^{-1} s_0 = \{c_0^*[\gamma - g(e_0^*)] + c_1^*\}[\gamma - g(e_0^*)]^{-1} p_0^h$$

As long as $p_1^h = p_0^h[\gamma - g(e_0^*)]^{-1}$*, the last equation becomes*

$$c_0^* p_0^h + c_1^* p_1^h = q_0^h s_0$$

And so, under

$$p_1^h = p_0^h[\gamma - g(e_0^*)]^{-1}$$
$$q_0^h = \gamma p_1^h$$

the socially efficient outcome is affordable. If we set $\lambda_1 = 1$*, then*

$$p_0^* = U'(c_0^*) < U'(c_0^{m*}) = p_0^{m*}$$

As for $p_1^* = p_0^*[\gamma - g(e_0^*)]^{-1}$*, the price at period* 1 *required so that the socially efficient outcome is affordable in a decentralised equilibrium, the effect is ambiguous because although* p_0^* *is lower than the prevailing price in the decentralised outcome,* $[\gamma - g(0)]^{-1} < [\gamma - g(e_0^*)]^{-1}$*. Hence,* p_1^* *is indeterminate, which in turn implies that* q_0^* *is also indeterminate. In other words, a* q_0 *supporting the socially efficient outcome necessarily implies a lower price of consumption in period* 0*. However, the effect* *on the price of period one can lower, higher or equal to the price prevailing in the competitive outcome and, hence, it is impossible to pin down what* q_0^* *would be relative to its competitive counterpart.*